CMPS COURSE FILE

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GEETHANJALI COLLEGE OF ENGINEERING AND TECHNOLOGY

DEPARTMENT OF *Electronics and Communication Engineering*

(Name of the Subject / Lab Course) : Electronic devices and circuits

(JNTU CODE -) **Programme : *UG***

Branch: EEE	**Version No : *05***
Year: III	**Updated on : 5/05/2013**
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4) Date :	**3) Design :**
	4) Date :

Approved by : (HOD) 1) Name :Dr.D.Radhika

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3) Date :

2. Syllabus:

Computer Methods & Power Systems

Subject Code:56011 L:4 T/P/D:0 Credits:4
Int. Marks:25 Ext. Marks:75 Total Marks:100

UNIT I: Power System Network Matrices-1

Graph Theory: Definitions, Bus Incidence Matrix, Y bus formation by Direct and Singular Transformation Methods, Numerical Problems.

UNIT II: Power System Network Matrices-2

Formation of ZBus: Partial network, Algorithm for the Modification of Z Bus Matrix for addition element for the following cases: Addition of element from a new bus to reference, Addition of element from a new bus to an old bus, Addition of element between an old bus to reference and Addition of element between two old busses (Derivations and Numerical Problems).- Modification of ZBus for the changes in network (Problems)

UNIT III: Power flow Studies-1

Necessity of Power Flow Studies – Data for Power Flow Studies – Derivation of Static load flow equations – Load flow solutions using Gauss Seidel Method: Acceleration Factor, Load flow solution with and without P-V buses, Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses): Determination of Bus Voltages, Injected Active and Reactive Powers (Sample One Iteration only) and finding Line Flows/Losses for the given Bus Voltages.

UNIT IV: Power flow Studies-2

Newton Raphson Method in Rectangular and Polar Co-Ordinates Form: Load Flow Solution with or without PV Busses- Derivation of Jacobian Elements, Algorithm and Flowchart. Decoupled and Fast Decoupled Methods. - Comparison of Different Methods – DC load Flow

UNIT V: Short Circuit Analysis-1

Per-Unit System of Representation. Per-Unit equivalent reactance network of a three phase Power System, Numerical Problems.

Symmetrical fault Analysis: Short Circuit Current and MVA Calculations, Fault levels, Application of Series Reactors, Numerical Problems.

UNIT VI: Short Circuit Analysis-2

Symmetrical Component Theory: Symmetrical Component Transformation, Positive, Negative and Zero sequence components: Voltages, Currents and Impedances.

Sequence Networks: Positive, Negative and Zero sequence Networks, Numerical Problems.

Unsymmetrical Fault Analysis: LG, LL, LLG faults with and without fault impedance, Numerical Problems.

UNIT VII: Power System Steady State Stability Analysis

Elementary concepts of Steady State, Dynamic and Transient Stabilities. Description of: Steady State Stability Power Limit, Transfer Reactance, Synchronizing Power Coefficient, Power Angle Curve and Determination of Steady State Stability and Methods to improve steady state stability.

UNIT VIII: Power System Transient State Stability Analysis

Derivation of Swing Equation. Determination of Transient Stability by Equal Area Criterion, Application of Equal Area Criterion, Critical Clearing Angle Calculation.- Solution of Swing Equation: Point-by-Point

Method. Methods to improve Stability - Application of Auto Reclosing and Fast Operating Circuit Breakers.

TEXT BOOKS:

1. power system Analysis Operation and control, Abhijit Chakrabarthi , Sunita Haldar, 3 ed , PHI,2010.
2. Modern Power system Analysis – by I.J.Nagrath & D.P.Kothari: Tata McGraw-Hill Publishing company, 2nd edition.

REFERENCE BOOKS:

1. Computer Techniques in Power System Analysis by M.A.Pai, TMH Publications
2. Power System Analysis by Grainger and Stevenson, Tata McGraw Hill.
3. Computer techniques and models in power systems, By K.Uma rao, I.K.International
4. Power System Analysis by Hadi Saadat – TMH Edition.

1. Vision of the department

Vision and Mission of the institute

The Mission of the institute

Our mission is to become a high quality premier educational institution, to create technocrats, by ensuring excellence, through enriched knowledge, creativity and self development.

The Vision of the institute

Geethanjali visualizes dissemination of knowledge and skills to students, who would eventually contribute to the well being of the people of the nation and global community.

DEPARTMENT OF EEE

Department of Electronics and Electronics Engineering is established in the year 2006 to meet the requirements of the Electrical and Electronic industries such as Vijay electrical, BHEL, BEL and society after the consultation with various stakeholders.

Vision of EEE

To provide excellent Electrical and electronics education by building strong teaching and research environment

2. Mission of the department

Mission of EEE

i) To offer high quality graduate program in Electrical and Electronics education and to prepare students for professional career or higher studies.
ii) The department promotes excellence in teaching, research, collaborative activities and positive contributions to society

3. PEOs and Pos

Program Educational Objectives

Program Educational Objectives of the UG Electrical and Electronics Engineering are:

PEO 1. Graduates will excel in professional career and/or higher education by acquiring knowledge in Mathematics, Science, Engineering principles and Computational skills.

PEO 2. Graduates will analyze real life problems, design Electrical systems appropriate to the requirement that are technically sound, economically feasible and socially acceptable.

PEO 3.Graduates will exhibit professionalism, ethical attitude, communication skills, team work in their profession, adapt to current trends by engaging in lifelong learning and participate in Research & Development.

Programme Outcomes

The Program Outcomes of UG in Electrical and Electronics Engineering are as follows:

PO 1. An ability to apply the knowledge of Mathematics, Science and Engineering in Electrical and Electronics Engineering.

PO 2. An ability to design and conduct experiments pertaining to Electrical and Electronics Engineering.

PO 3. An ability to function in multidisciplinary teams

PO 4. An ability to simulate and determine the parameters such as nominal voltage, current, power and associated attributes.

PO 5. An ability to identify, formulate and solve problems in the areas of Electrical and Electronics Engineering.

PO 6. An ability to use appropriate network theorems to solve electrical engineering problems.

PO 7. An ability to communicate effectively.

PO 8. An ability to visualize the impact of electrical engineering solutions in global, economic and societal context.

PO 9. Recognition of the need and an ability to engage in life-long learning.

PO 10 An ability to understand contemporary issues related to alternate energy sources.

PO 11 An ability to use the techniques, skills and modern engineering tools necessary for Electrical Engineering Practice.
PO 12 An ability to simulate and determine the parameters like voltage profile and current ratings of transmission lines in Power Systems.
PO 13 An ability to understand and determine the performance of electrical machines namely speed, torque, efficiency etc.
PO 14 An ability to apply electrical engineering and management principles to Power Projects

6. Course objectives and outcomes:

Objectives:

- This course introduces formation of Z bus of a transmission line, power flow studies by various methods.
- It also deals with short circuit analysis and analysis of power system for steady state and transient stability.

Outcomes

On successful completion of this subject, students will be able to:

1. Demonstrate an understanding of the nature of the modern power system, including the behaviour of the constituent components and sub-systems
2. Describe the construction, operation and equivalent circuit of three-phase transformers
3. Apply load flow analysis to an electrical power network and interpret the results of the analysis
4. Analyse a network under both balanced and unbalanced fault conditions and interpret the results
5. Demonstrate an understanding of the role of protection in modern power systems and to describe the operation of a range of protection schemes
6. Design a protection system for an item of electrical plant
7. Demonstrate an awareness of the methods used for voltage regulation in electrical power networks
8. Analyse the transient stability of a single machine/infinite bus system using both analytical and time simulation methods
9. Demonstrate an understanding of the factors which determine transient stability in both single machine and multi-machine systems
10. Describe the role of insulation co-ordination in the design and operation of power networks, including the role of circuit breakers
11. Demonstrate the ability to conduct experiments in the Electrical Engineering Laboratory in accordance with Health and Safety Regulations and to record, interpret and report on the experimental results

7. Brief notes on the importance of the course and how it fit into the curriculum

The course is about the basic concept, mathematical model, analysis method of power systems. It covers power flow calculation, fault calculation, stability analysis, control and protection for power systems. Matrix analysis of power systems networks and methods of solution. Load flow and short circuit analysis. Economic operation of power systems. Transient stability analysis.

8. Prerequisites

Principles of Electric Circuits, Electrical Machine

9. Instructional Learning Outcomes:

Outcomes

1. Demonstrate an understanding of the nature of the modern power system, including the behaviour of the constituent components and sub-systems
2. Describe the construction, operation and equivalent circuit of three-phase transformers
3. Apply load flow analysis to an electrical power network and interpret the results of the analysis
4. Analyse a network under both balanced and unbalanced fault conditions and interpret the results
5. Demonstrate an understanding of the role of protection in modern power systems and to describe the operation of a range of protection schemes
6. Design a protection system for an item of electrical plant
7. Demonstrate an awareness of the methods used for voltage regulation in electrical power networks
8. Analyse the transient stability of a single machine/infinite bus system using both analytical and time simulation methods
9. Demonstrate an understanding of the factors which determine transient stability in both single machine and multi-machine systems
10. Describe the role of insulation co-ordination in the design and operation of power networks, including the role of circuit breakers
11. Demonstrate the ability to conduct experiments in the Electrical Engineering Laboratory in accordance with Health and Safety Regulations and to record, interpret and report on the experimental results

10.Course mapping with PEOs and Pos:

a) an ability to apply the knowledge of Mathematics, science and engineering in Electronics and communications	
b) an ability to Design & Conduct Experiments, as well as analyze & Interpret Data	√
c) an ability to design a system, component, or process to meet desired needs with in realistic constraints such as economic, environmental, social, political, ethical, health and safety, manufacturability, and sustainability	√
d) an ability to function on multidisciplinary teams	√
e) an ability to Identify, Formulate & Solve problems in the area of Electronics and Communications Engineering	√
f) an understanding of professional and ethical responsibility	
g) an ability to communicate effectively	
h) the broad education necessary to understand the impact of engineering solutions in a global, economic, environmental, and societal context	
i) a recognition of the need for, and an ability to engage in life-long learning	√
j) a knowledge of contemporary issues	
k) an ability to use the techniques, skills, and modern engineering tools necessary for engineering practice	√

Relationship of the course to the program educational objectives :

1. **Domain knowledge:** Graduates will be able to synthesize mathematics, science, engineering fundamentals, laboratory and work-based experiences to formulate and solve engineering problems in Electronics and Communication engineering domains and shall have proficiency in Computer-based engineering and the use of computational tools.	√
2. **Professional Employment:** Graduates will succeed in entry-level engineering positions within the core Electronics and Communication Engineering,	√

computational or manufacturing firms in regional, national, or international industries and with government agencies.	
3. **Higher Degrees:** Graduates will succeed in the pursuit of advanced degrees in Engineering or other fields where a solid foundation in mathematics, science, and engineering fundamentals is required.	√
4. **Engineering citizenship:** Graduates will be prepared to communicate and work effectively on team based engineering projects and will practice the ethics of their profession consistent with a sense of social responsibility.	
5. **Lifelong Learning:** Graduates will recognize the importance of, and have the skills for, continued independent learning to become experts in their chosen fields and to broaden their professional knowledge.	√
6. **Research and Development:** To undertake Research and Development works in the areas of Electronics and Communication fields.	√

11. Class Time Table:

12. Individual Time Table

13. Lecture schedule with methodology being used/adopted

Unit wise Summary

S. NO	Unit No	Date	Topic	Regular/ Additional	LCD/ OHP/BB	Remark
			UNIT-1 Power System Network Matrices-1			
1	1		Graph Theory,Definitions			
2			Bus Incidence Matrix			
3			Y bus Formation by Direct Method		OHP	
4			Y bus Formation by Singular Transformation Method			
5			Numerical Problems			
			UNIT-2 Power System Network Matrices-2			
6	2		Formation of Z Bus			
7			Algorithm for the Modification of Z Bus Addition of Element from a New Bus to Reference		LCD	
8			Addition of -Element from a New Bus to an Old Bus			
9			Addition of Element between Old Buses to Reference			
10			Addition of Element between Two Old Buses			
11			Modifications of Z Bus for the changes in Network Numerical Problems			
12			TOTURIAL 1			
13			TOTURIAL 2			
14			ASSGHNMENT FOR 1 & 2 UNITS			
			UNIT-3 Power flow Studies-1			
15	3		Necessity of Power Flow Studies			
16			Data for Power Flow Studies			
17			Derivation of Static Load Flow Equations			
18			Gauss Seidel Method, Acceleration Factor		OHP	
19			Load Flow Solution with and without P-V Buses		OHP	
20			Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses):		OHP	
21			Determination of Bus Voltages			
22			Injected Active and Reactive Powers			
23			Finding Live Flows / Losses foe given Bus Voltages.			
			Unit-4 Power flow Studies-2			

24	4		Newton Raphson Method in Rectangular and Polar Co-Ordinates Form:		OHP	
25			Load Flow Solution with or without PV Busses-		OHP	
26			Derivation of Jacobean Elements Algorithm and Flowchart.			
27			Decoupled and Fast Decoupled Methods			
28			Comparison of Different Methods – DC load Flow			
29			TOTURIAL 1			
30			TOTURIAL 2			
31			ASSGHNMENT FOR 1 & 2 UNITS			
32			I Mid revision			
			Unit-5 Short Circuit Analysis-1			
33	5		Per-Unit System of Representation.			
34			Per-Unit equivalent reactance network of a three phase Power System, Numerical Problems.			
35			Symmetrical fault Analysis: Short Circuit Current and MVA Calculations			
36			Fault levels, Application of Series Reactors			
37			Numerical Problems.			
			UNIT-6 Short Circuit Analysis-2			
38	6		Symmetrical Component Theory: Symmetrical Component ransformation			
39			Positive, Negative and Zero sequence components: Voltages, Currents and Impedances.			
40			Sequence Networks: Positive, Negative and Zero sequence Networks, Numerical Problems.			
41			Unsymmetrical Fault Analysis: LG, LL, LLG faults with and without fault impedance			
42			Numerical Problems.			
43			TOTURIAL 1			
44			TOTURIAL 2			
45			ASSGHNMENT FOR 1 & 2 UNITS			
			UNIT-7 Power System Steady State Stability Analysis			
46	7		Elementary concepts of Steady State, Dynamic and Transient Stabilities.			
47			Description of: Steady State Stability Power		OHP	

			Limit, Transfer Reactance			
48			Synchronizing Power Coefficient, Power Angle Curve			
49			Determination of Steady State Stability and Methods to improve steady state stability.			
			Unit-8 Power System Transient State Stability Analysis			
50	8		Derivation of Swing Equation. Determination of Transient Stability by Equal Area Criterion		OHP	
51			Application of Equal Area Criterion, Critical Clearing Angle Calculation			
52			Solution of Swing Equation: Point-by-Point Method.			
53			Methods to improve Stability			
54			Application of Auto Reclosing and Fast Operating Circuit Breakers.		OHP	
55			TOTURIAL 1			
56			TOTURIAL 2			
57			ASSGHNMENT FOR 1 & 2 UNITS			
58			II Mid revision			
A1				**additional**		
A2				**additional**		
			Revision for previsious JNTU papers			
			Revision for previsious JNTU papers			
			Revision for previsious JNTU papers			
			Revision for previsious JNTU papers			
			TOTAL NUMBER OF PERIODS=64+2(ADDITIONAL)			

MICRO PLAN

S. NO	**Unit No**	**Date**	**Topic**	**Regular/ Additional**	**LCD/ OHP/BB**	**Remark**
			UNIT-1 Power System Network Matrices-1			
1	1		Graph Theory,Definitions			
2			Bus Incidence Matrix			
3			Y bus Formation by Direct Method		OHP	
4			Y bus Formation by Singular Transformation Method			

5			Numerical Problems			
			UNIT-2 Power System Network Matrices-2			
6	2		Formation of Z Bus			
7			Algorithm for the Modification of Z Bus Addition of Element from a New Bus to Reference		LCD	
8			Addition of -Element from a New Bus to an Old Bus			
9			Addition of Element between Old Buses to Reference			
10			Addition of Element between Two Old Buses			
11			Modifications of Z Bus for the changes in Network Numerical Problems			
12			TOTURIAL 1			
13			TOTURIAL 2			
14			ASSGHNMENT FOR 1 & 2 UNITS			
			UNIT-3 Power flow Studies-1			
15	3		Necessity of Power Flow Studies			
16			Data for Power Flow Studies			
17			Derivation of Static Load Flow Equations			
18			Gauss Seidel Method, Acceleration Factor		OHP	
19			Load Flow Solution with and without P-V Buses		OHP	
20			Algorithm and Flowchart. Numerical Load flow Solution for Simple Power Systems (Max. 3-Buses):		OHP	
21			Determination of Bus Voltages			
22			Injected Active and Reactive Powers			
23			Finding Live Flows / Losses foe given Bus Voltages.			
			Unit-4 Power flow Studies-2			
24	4		Newton Raphson Method in Rectangular and Polar Co-Ordinates Form:		OHP	
25			Load Flow Solution with or without PV Busses-		OHP	
26			Derivation of Jacobean Elements Algorithm and Flowchart.			
27			Decoupled and Fast Decoupled Methods			
28			Comparison of Different Methods – DC load Flow			
29			TOTURIAL 1			
30			TOTURIAL 2			
31			ASSGHNMENT FOR 1 & 2 UNITS			
32			I Mid revision			

			Unit-5 Short Circuit Analysis-1			
33	5		Per-Unit System of Representation.			
34			Per-Unit equivalent reactance network of a three phase Power System, Numerical Problems.			
35			Symmetrical fault Analysis: Short Circuit Current and MVA Calculations			
36			Fault levels, Application of Series Reactors			
37			Numerical Problems.			
			UNIT-6 Short Circuit Analysis-2			
38	6		Symmetrical Component Theory: Symmetrical Component ransformation			
39			Positive, Negative and Zero sequence components: Voltages, Currents and Impedances.			
40			Sequence Networks: Positive, Negative and Zero sequence Networks, Numerical Problems.			
41			Unsymmetrical Fault Analysis: LG, LL, LLG faults with and without fault impedance			
42			Numerical Problems.			
43			TOTURIAL 1			
44			TOTURIAL 2			
45			ASSGHNMENT FOR 1 & 2 UNITS			
			UNIT-7 Power System Steady State Stability Analysis			
46	7		Elementary concepts of Steady State, Dynamic and Transient Stabilities.			
47			Description of: Steady State Stability Power Limit, Transfer Reactance		OHP	
48			Synchronizing Power Coefficient, Power Angle Curve			
49			Determination of Steady State Stability and Methods to improve steady state stability.			
			Unit-8 Power System Transient State Stability Analysis			
50	8		Derivation of Swing Equation. Determination of Transient Stability by Equal Area Criterion		OHP	
51			Application of Equal Area Criterion, Critical Clearing Angle Calculation			
52			Solution of Swing Equation: Point-by-Point Method.			

53			Methods to improve Stability			
54			Application of Auto Reclosing and Fast Operating Circuit Breakers.		OHP	
55			TOTURIAL 1			
56			TOTURIAL 2			
57			ASSGHNMENT FOR 1 & 2 UNITS			
58			II Mid revision			
A1				**additional**		
A2				**additional**		
			Revision for previsious JNTU papers			
			Revision for previsious JNTU papers			
			Revision for previsious JNTU papers			
			Revision for previsious JNTU papers			
			TOTAL NUMBER OF PERIODS=64+2(ADDITIONAL)			

13.8. Subject Contents

13.8. 1. Synopsis page for each period(62 pages)

13.8.2. Detailed Lecture notes containing:

1.ppts
2.ohp slides
3.subjective type questions(approximately 5 t0 8 in no)
4.objective type questions(approximately 20 to 30 in no)
5.Any simulations

13.9. Course Review (By the concerned Faculty):

(I)Aims
(II) Sample check
(III) End of the course rreport by the concerned faculty

GUIDELINES:

Distribution of periods :

No. of classes required to cover JNTU syllalbus : 45
No. of classes required to cover Additional topics : 5
No. of classes required to cover Assignment tests (for every 1 units 1 test) : 5
No. of classes required to cover tutorials : 5
No. of classes required to cover Mid tests : 2
No of classes required to solve University : 5
Question papers -------
Total periods 67
Total periods 67

14. Detailed notes:

CHAPTER- 1-A

INCIDENCE AND NETWORK MATRICES

[**CONTENTS:** Definitions of important terms, Incidence matrices: Element node incidence matrix and Bus incidence matrix, Primitive networks and matrices, Performance of primitive networks, Frames of reference, Singular transformation analysis, Formation of bus admittance matrix, examples]

INTRODUCTION

The solution of a given linear network problem requires the formation of a set of equations describing the response of the network. The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components. In the bus frame of reference the variables are the node voltages and node currents.

The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix. The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems. The formulation of the network equations in different frames of reference requires the knowledge of graph theory. Elementary graph theory concepts are presented here, followed by development of network equations in the bus frame of reference.

ELEMENTARY LINEAR GRAPH THEORY: IMPORTANT TERMS

The geometrical interconnection of the various branches of a network is called the *topology* of the network. The connection of the network topology, shown by replacing all its elements by lines is called a *graph*. A *linear graph* consists of a set of objects called *nodes* and another set called *elements* such that each element is identified with an ordered pair of nodes. An *element* is defined as any line segment of the graph irrespective of the characteristics of the components involved. A graph in which a

direction is assigned to each element is called an *oriented graph* or a *directed graph*. It is to be noted that the directions of currents in various elements are arbitrarily assigned and the network equations are derived, consistent with the assigned directions. Elements are indicated by numbers and the nodes by encircled numbers. The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph : This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1, with:

e = number of elements = 6	l = number of links = e-b = 3
n = number of nodes = 4	Tree = T(1,2,3) and
b = number of branches = n-1 = 3	Co-tree = T(4,5,6)

Sub-graph : sG is a sub-graph of G if the following conditions are satisfied:

- sG is itself a graph
- Every node of sG is also a node of G
- Every branch of sG is a branch of G

For eg., sG(1,2,3), sG(1,4,6), sG(2), sG(4,5,6), sG(3,4),.. are all valid sub-graphs of the oriented graph of Fig.1c.

Loop : A sub-graph L of a graph G is a loop if

- L is a connected sub-graph of G
- Precisely two and not more/less than two branches are incident on each node in L

In Fig 1c, the set{1,2,4} forms a loop, while the set{1,2,3,4,5} is not a valid, although the set(1,3,4,5) is a valid loop. The KVL (Kirchhoff's Voltage Law) for the loop is stated as follows: *In any lumped network, the algebraic sum of the branch voltages around any of the loops is zero.*

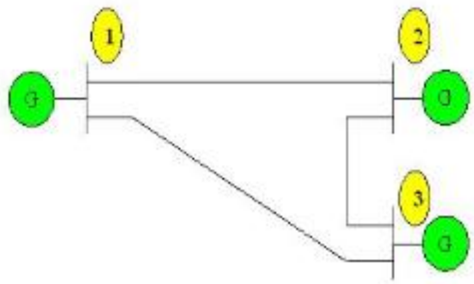


Fig 1a. Single line diagram of a power system

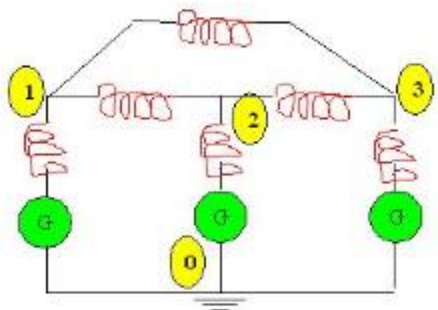


Fig 1b. Reactance diagram

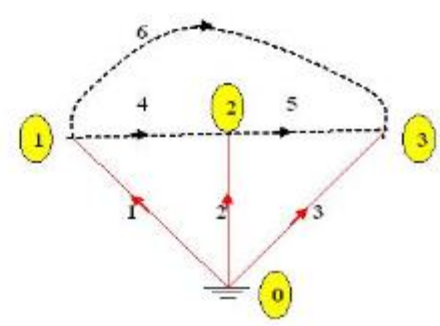


Fig 1c. Oriented Graph

Cutset : It is a set of branches of a connected graph G which satisfies the following conditions :

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1c, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected sub-graphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: *In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.*

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with n nodes,

$$\textbf{The number of branches: } \mathbf{b} = n-1 \quad \textbf{(1)}$$

For the graph of Fig 1c, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the *co-tree*. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called *links*. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

$$\textbf{The number of links: } l = e - b = e - n + 1 \quad \textbf{(2)}$$

For the graph of Fig 1c, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

Examples on Basics of LG Theory:

Example-1: *Obtain the oriented graph for the system shown in Fig. E1. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.*

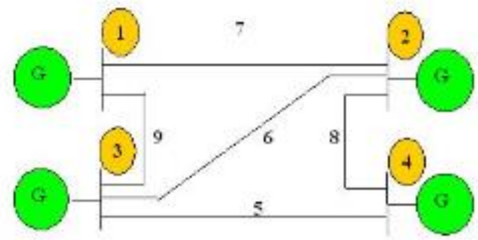


Fig. E1a. Single line diagram of Example System

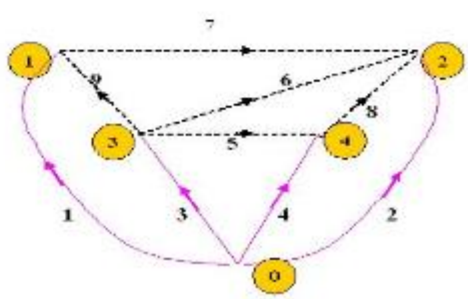


Fig. E1b. Oriented Graph of Fig. E1a.

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cut-sets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figure E1c and Fig.E1d respectively.

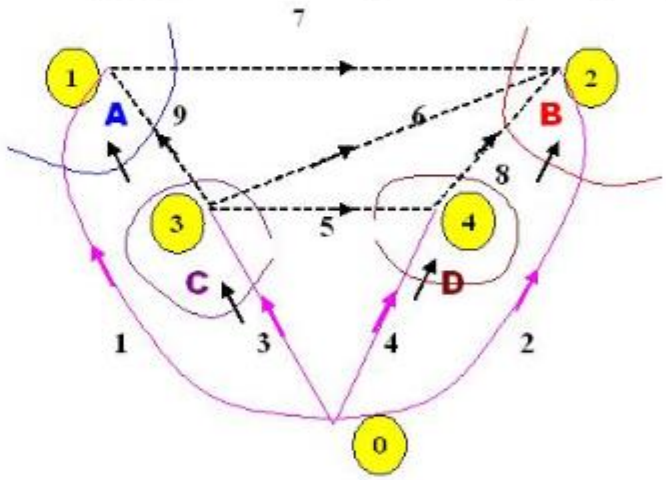


Fig. E1c. Basic Cutsets of Fig. E1a.

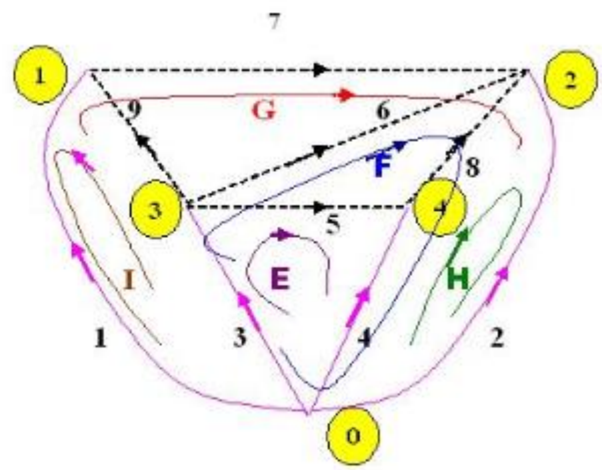


Fig. E1d. Basic Loops of Fig. E1a.

INCIDENCE MATRICES

Element–node incidence matrix: $\hat{A}$

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, $\hat{A}$. An element a_{ij} of $\hat{A}$ is defined as under:

a_{ij} = 1 if the branch-i is incident to and oriented away from the node-j.

= -1 if the branch-i is incident to and oriented towards the node-j.

= 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of $\hat{A}$ is $e \times n$, where e is the number of elements and n is the number of nodes in the network. For example, consider again the sample system with its oriented graph as in fig. 1c. the corresponding element-node incidence matrix, is obtained as under:

$\hat{A}$ =

Nodes / Elements	0	1	2	3
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1

It is to be noted that the first column and first row are not part of the actual matrix and they only indicate the element number node number respectively as shown. Further, the sum of every row is found to be equal to zero always. Hence, the rank of the matrix is less than n. Thus in general, the matrix $\hat{A}$ satisfies the identity:

$$\sum_{j=1}^{n} a_{ij} = 0 \quad \forall \; i = 1,2,\ldots..e. \tag{3}$$

Bus incidence matrix: A

By selecting any one of the nodes of the connected graph as the reference node, the corresponding column is deleted from $\hat{A}$ to obtain the bus incidence matrix, A. The dimensions of A are $e\times(n\text{-}1)$ and the rank is $n\text{-}1$. In the above example, selecting node-0 as reference node, the matrix A is obtained by deleting the column corresponding to node-0, as under:

A =

Buses / Elements	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

It may be observed that for a selected tree, say, T(1,2,3), the bus incidence matrix can be so arranged that the branch elements occupy the top portion of the A-matrix followed by the link elements. Then, the matrix-A can be partitioned into two sub matrices A_b and A_l as shown, where,

(i) A_b is of dimension (bxb) corresponding to the branches and

(ii) A_l is of dimension (lxb) corresponding to links.

A is a rectangular matrix, hence it is singular. A_b is a non-singular square matrix of dimension-b. Since A gives the incidence of various elements on the nodes with their direction of incidence, the KCL for the nodes can be written as

$$A^T \bar{i} = 0 \qquad \textbf{(4)}$$

where A^T is the transpose of matrix A and $\bar{i}$ is the vector of branch currents. Similarly for the branch voltages we can write,

$$\bar{v} = A \ \bar{E}_{bus} \qquad \textbf{(5)}$$

Examples on Bus Incidence Matrix:

Example-2: *For the sample network-oriented graph shown in Fig. E2, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and* $\hat{A}$. *Also show the partitioned form of the matrix-A.*

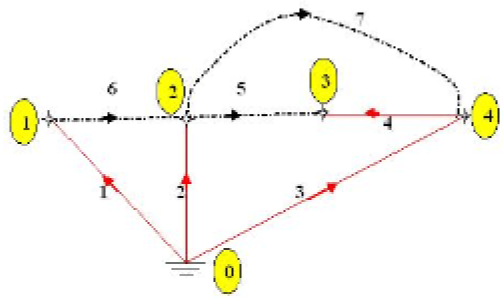


Fig. E2. Sample Network-Oriented Graph

$$\hat{A} = \text{Elements} \begin{array}{c} \\ \\ \end{array} \overset{\text{nodes}}{\begin{bmatrix} e\backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

$$\mathbf{A} = \text{Elements} \overset{\text{buses}}{\begin{bmatrix} e\backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$$A_b = \text{branches} \begin{array}{c} \text{buses} \\ \begin{bmatrix} b \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix} \end{array}$$

$$A_l = \text{links} \begin{array}{c} \text{buses} \\ \begin{bmatrix} l \backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Example-3: *For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and $\hat{A}$. Also show the partitioned form of the matrix-A.*

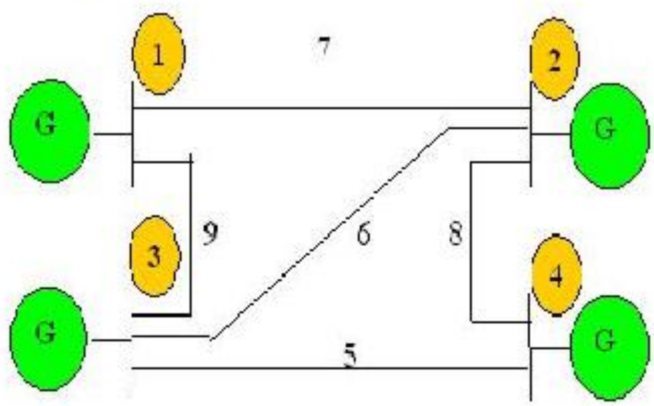


Fig. E3a. Sample Example network

Consider the oriented graph of the given system as shown in figure E3b, below.

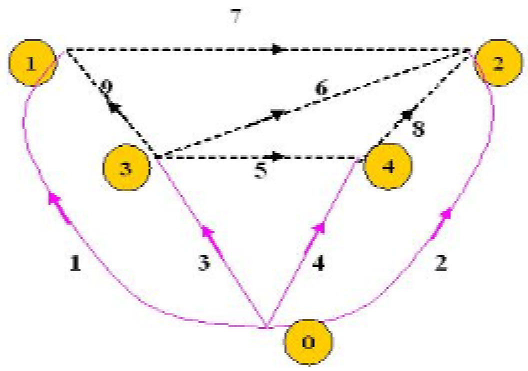


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices □ and A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two sub-matrices as under:

$\mathbf{A_b}$ =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

$\mathbf{A_l}$ =

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

PRIMITIVE NETWORKS

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network. The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices. An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component.

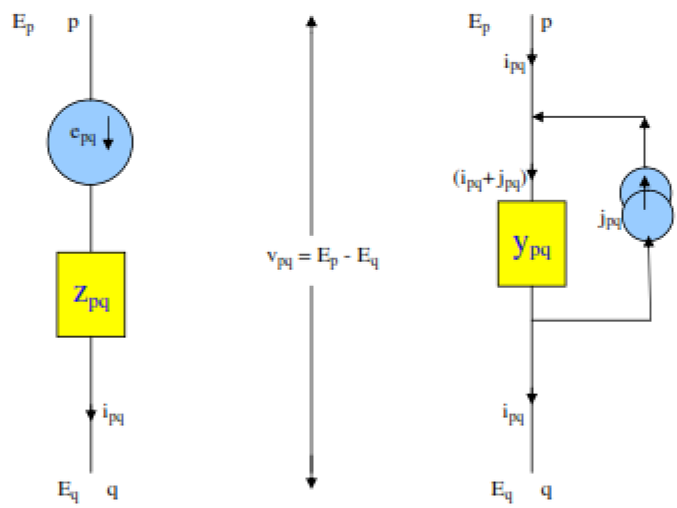


Fig.2 Representation of a primitive network element

(a) Impedance form (b) Admittance form

The network performance can be represented by using either the impedance or the admittance form of representation. With respect to the element, p-q, let,

v_{pq} = voltage across the element p-q,

e_{pq} = source voltage in series with the element p-q,

i_{pq}= current through the element p-q,

j_{pq}= source current in shunt with the element p-q,

z_{pq}= self impedance of the element p-q and

y_{pq}= self admittance of the element p-q.

Performance equation: Each element p-q has two variables, v_{pq} and i_{pq}. The performance of the given element p-q can be expressed by the performance equations as under:

$v_{pq} + e_{pq} = z_{pq}i_{pq}$ (in its impedance form)

$i_{pq} + j_{pq} = y_{pq}v_{pq}$ (in its admittance form) **(6)**

Thus the parallel source current jpq in admittance form can be related to the series source voltage, epq in impedance form as per the identity:

$j_{pq} = - y_{pq} e_{pq}$ **(7)**

A set of non-connected elements of a given system is defined as a *primitive Network and a*n element in it is a fundamental element that is not connected to any other element. In the equations above, if the variables and parameters are replaced by the corresponding vectors and matrices, referring to the complete set of elements present in a given system, then, we get the performance equations of the primitive network in the form as under:

$v + e = [z]\, i$

$i + j = [y]\, v$ **(8)**

Primitive network matrices:

A diagonal element in the matrices, [z] or [y] is the self impedance $z_{pq\text{-}pq}$ or self admittance, $y_{pq\text{-}pq}$. An off-diagonal element is the mutual impedance, $z_{pq\text{-}rs}$ or mutual admittance, $y_{pq\text{-}rs}$, the value present as a mutual coupling between the elements p-q and r-s. The primitive network admittance matrix, [y] can be obtained also by inverting the primitive impedance matrix, [z]. Further, if there are no mutually coupled elements in the given system, then both the matrices, [z] and [y] are diagonal. In such cases, the self impedances are just equal to the reciprocal of the corresponding values of self admittances, and vice-versa.

Examples on Primitive Networks:

Example-4: *Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: $Z_1=Z_2=0.2$, $Z_3=0.25$, $Z_4=Z_5=0.1$ and $Z_6=0.4$ units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.*

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Solution:

The element node incidence matrix, $\hat{A}$ can be obtained from the given A matrix, by pre-augmenting to it an extra column corresponding to the reference node, as under.

$\hat{A}$ =

1	-1	0	0
1	0	-1	0
1	0	0	-1
0	1	-1	0
0	0	1	-1
0	1	0	-1

Based on the conventional definitions of the elements of $\hat{A}$, the oriented graph can be formed as under:

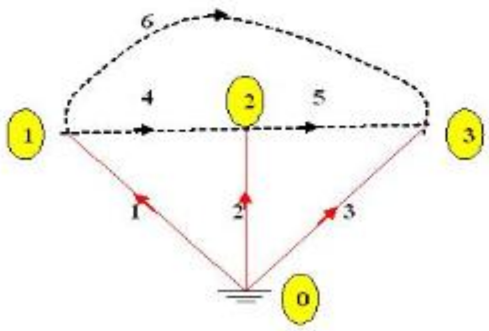


Fig. E4 Oriented Graph

Thus the primitive network matrices are square, symmetric and diagonal matrices of order e=no. of elements = 6. They are obtained as follows.

[z] =

0.2	0	0	0	0	0
0	**0.2**	0	0	0	0
0	0	**0.25**	0	0	0
0	0	0	**0.1**	0	0
0	0	0	0	**0.1**	0
0	0	0	0	0	**0.4**

And

[y] =

5.0	0	0	0	0	0
0	**5.0**	0	0	0	0
0	0	**4.0**	0	0	0
0	0	0	**10**	0	0
0	0	0	0	**10**	0
0	0	0	0	0	**2.5**

Example-5: *Consider three passive elements whose data is given in Table E5 below. Form the primitive network impedance matrix.*

Table E5

Element number	Self impedance ($z_{pq\text{-}pq}$)		Mutual impedance, ($z_{pq\text{-}rs}$)	
	Bus-code, (p-q)	**Impedance in p.u.**	**Bus-code, (r-s)**	**Impedance in p.u.**
1	1-2	j 0.452		
2	2-3	j 0.387	1-2	j 0.165
3	1-3	j 0.619	1-2	j 0.234

Solution:

$$[z] = \begin{array}{c|ccc|} & \text{1-2} & \text{2-3} & \text{1-3} \\ \hline \text{1-2} & j\,0.452 & j\,0.165 & j\,0.234 \\ \text{2-3} & j\,0.165 & j\,0.387 & 0 \\ \text{1-3} & j\,0.234 & 0 & j\,0.619 \\ \hline \end{array}$$

Note:

- The size of [z] is $e \times e$, where e= number of elements,
- The diagonal elements are the self impedances of the elements
- The off-diagonal elements are mutual impedances between the corresponding elements.
- Matrices [z] and [y] are inter-invertible.

FORMATION OF Y_{BUS} AND Z_{BUS}

The bus admittance matrix, Y_{BUS} plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

1. Rule of Inspection
2. Singular Transformation
3. Non-Singular Transformation
4. Z_{BUS} Building Algorithms, etc.

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS}\, I_{BUS}$$

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad \textbf{(9)}$$

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR}\, I_{BR}$$

$$I_{BR} = Y_{BR}\, E_{BR} \qquad \textbf{(10)}$$

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP}\, I_{LOOP}$$

$$I_{LOOP} = Y_{LOOP}\, E_{LOOP} \qquad \textbf{(11)}$$

Of the various network matrices refered above, the bus admittance matrix (Y_{BUS}) and the bus impedance matrix (Z_{BUS}) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3(V_1-V_3) + Y_6(V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5(V_2-V_3) + Y_6(V_2 - V_1)$

At node 3: $0 = Y_3(V_3-V_1) + Y_4V_3 + Y_5(V_3 - V_2)$ **(12)**

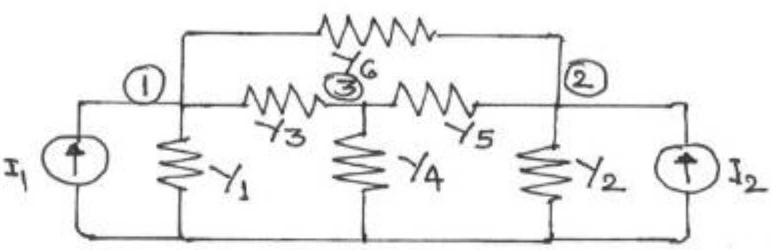


Fig. 3 Example System for finding Y_{BUS}

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad \textbf{(13)}$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS} E_{BUS} \qquad \textbf{(14)}$$

Where, Y_{BUS} is the bus admittance matrix, I_{BUS} & E_{BUS} are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, Y_{BUS} of equation (13), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Y_{ii}) of the bus admittance matrix, Y_{BUS}, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Y_{ij}) of the bus admittance matrix, Y_{BUS}, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots.n)$$
$$Y_{ij} = -\, y_{ij} \quad (j = 1,2,\ldots\ldots.n) \qquad \textbf{(15)}$$

For $i = 1,2,\ldots,n$, n = no. of buses of the given system, y_{ij} is the admittance of element connected between buses i and j and y_{ii} is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix

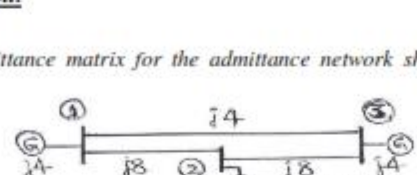
In cases where, the bus impedance matrix is also required, it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible.

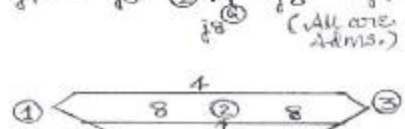
Note: It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

Examples on Rule of Inspection:

Example 6: *Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection*

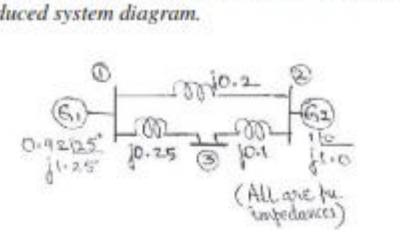
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



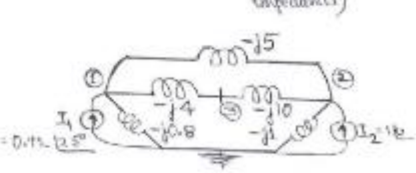


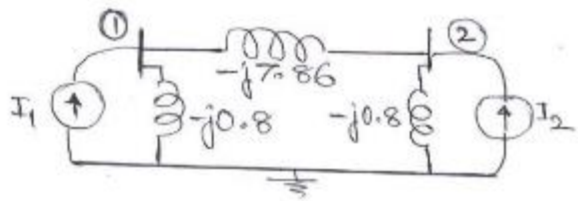
Example 7: *Obtain Y_{BUS} for the impedance network shown aside by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*

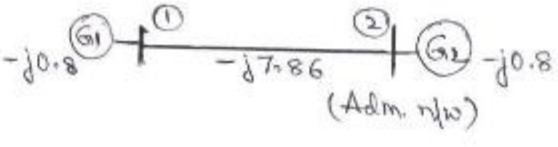
$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$



$$Z_{BUS} = Y_{BUS}^{-1}$$







$$\mathbf{Y_{BUS}^{New} = Y_A - Y_B Y_D^{-1} Y_C}$$

$$\mathbf{Y_{BUS} = j\begin{vmatrix} -8.66 & 7.86 \\ 7.86 & -8.66 \end{vmatrix}}$$

SINGULAR TRANSFORMATIONS

The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, Y_{BUS} and Bus impedance matrix, Z_{BUS}

In the bus frame of reference, the performance of the interconnected network is described by n independent nodal equations, where n is the total number of buses (n+1 nodes are present, out of which one of them is designated as the reference node). For example a 5-bus system will have 5 external buses and 1 ground/ ref. bus). The performance equation relating the bus voltages to bus current injections in bus frame of reference in admittance form is given by

$$I_{BUS} = Y_{BUS}\, E_{BUS} \tag{17}$$

Where E_{BUS} = vector of bus voltages measured with respect to reference bus

I_{BUS} = Vector of currents injected into the bus

Y_{BUS} = bus admittance matrix

The performance equation of the primitive network in admittance form is given by

$$i + j = [y]\, v$$

Pre-multiplying by A^t (transpose of A), we obtain

$$A^t i + A^t j = A^t [y]\, v \tag{18}$$

However, as per equation (4),

$$A^t i = 0,$$

since it indicates a vector whose elements are the algebraic sum of element currents incident at a bus, which by Kirchhoff's law is zero. Similarly, $A^t j$ gives the algebraic sum of all source currents incident at each bus and this is nothing but the total current injected at the bus. Hence,

$$A^t j = I_{BUS} \tag{19}$$

Thus from (18) we have, $\quad I_{BUS} = A^t [y]\, v \qquad$ **(20)**

However, from (5), we have

$$v = A\, E_{BUS}$$

And hence substituting in (20) we get,

$$I_{BUS} = A^t [y]\, A\, E_{BUS} \tag{21}$$

Comparing (21) with (17) we obtain,

$$Y_{BUS} = A^t [y]\, A \tag{22}$$

The bus incidence matrix is rectangular and hence singular. Hence, (22) gives a singular transformation of the primitive admittance matrix [y]. The bus impedance matrix is given by ,

$$Z_{BUS} = Y_{BUS}^{-1} \tag{23}$$

Note: This transformation can be derived using the concept of power invariance, however, since the transformations are based purely on KCL and KVL, the transformation will obviously be power invariant.

Examples on Singular Transformation:

Example 8: *For the network of Fig E8, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table E8.*

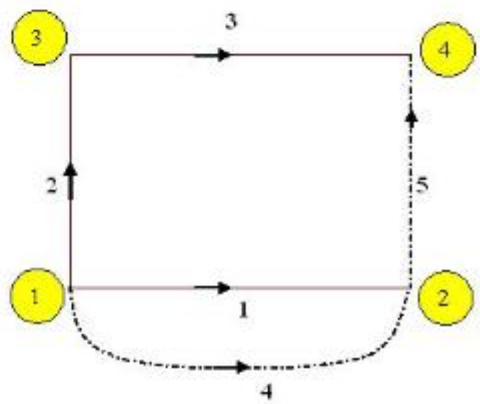


Fig E8 System for Example-8

Table E8: Data for Example-8

Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

Solution:

The bus incidence matrix is formed taking node 1 as the reference bus.

$$A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & -1 \\ -1 & 0 & 0 \\ 1 & 0 & -1 \end{bmatrix}$$

The primitive incidence matrix is given by,

$$[z] = \begin{bmatrix} j0.6 & j0.1 & 0.0 & j0.2 & 0.0 \\ j0.1 & j0.5 & 0.0 & 0.0 & 0.0 \\ 0.0 & 0.0 & j0.5 & 0.0 & 0.0 \\ j0.2 & 0.0 & 0.0 & j0.4 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & j0.2 \end{bmatrix}$$

The primitive admittance matrix $[y] = [z]^{-1}$ and given by,

$$[y] = \begin{bmatrix} -j2.0833 & j0.4167 & 0.0 & j1.0417 & 0.0 \\ j0.4167 & -j2.0833 & 0.0 & -j0.2083 & 0.0 \\ 0.0 & 0.0 & -j2.0 & 0.0 & 0.0 \\ j1.0417 & -j0.2083 & 0.0 & -j3.0208 & 0.0 \\ 0.0 & 0.0 & 0.0 & 0.0 & -j5.0 \end{bmatrix}$$

The bus admittance matrix by singular transformation is obtained as

$$Y_{BUS} = A^t\,[y]\,A = \begin{bmatrix} -j8.0208 & j0.2083 & j5.0 \\ j0.2083 & -j4.0833 & j2.0 \\ j5.0 & j2.0 & -j7.0 \end{bmatrix}$$

$$Z_{BUS} = Y_{BUS}^{-1} = \begin{bmatrix} j0.2713 & j0.1264 & j0.2299 \\ j0.1264 & j0.3437 & j0.1885 \\ j0.2299 & j0.1885 & j0.3609 \end{bmatrix}$$

SUMMARY

The formulation of the mathematical model is the first step in obtaining the solution of any electrical network. The independent variables can be either currents or voltages. Correspondingly, the elements of the coefficient matrix will be impedances or admittances.

Network equations can be formulated for solution of the network using graph theory, independent of the nature of elements. In the graph of a network, the tree-branches and links are distinctly identified. The complete information about the interconnection of the network, with the directions of the currents is contained in the bus incidence matrix.

The information on the nature of the elements which form the interconnected network is contained in the primitive impedance matrix. A primitive element can be represented in impedance form or admittance form. In the bus frame of reference, the performance of the interconnected system is described by $(n-1)$ nodal equations, where n is the number of nodes. The bus admittance matrix and the bus impedance matrix relate the bus voltages and currents. These matrices can be obtained from the primitive impedance and admittance matrices.

Chapter-1-B

FORMATION OF BUS IMPEDANCE MATRIX

[CONTENTS: *Node elimination by matrix algebra, generalized algorithms for Z_{BUS} building, addition of BRANCH, addition of LINK, special cases of analysis, removal of elements, changing the impedance value of an element, examples*]

NODE ELIMINATION BY MATRIX ALGEBRA

Nodes can be eliminated by the matrix manipulation of the standard node equations. However, *only those nodes at which current does not enter or leave the network can be considered for such elimination*. Such nodes can be eliminated either in one group or by taking the eligible nodes one after the other for elimination, as discussed next.

CASE-A: Simultaneous Elimination of Nodes:

Consider the performance equation of the given network in bus frame of reference in admittance form for a n-bus system, given by:

$$\mathbf{I_{BUS} = Y_{BUS}\,E_{BUS}} \qquad \textbf{(1)}$$

Where I_{BUS} and E_{BUS} are n-vectors of injected bus current and bus voltages and Y_{BUS} is the square, symmetric, coefficient bus admittance matrix of order n.

Now, of the n buses present in the system, let p buses be considered for node-elimination so that the reduced system after elimination of p nodes would be retained with m (= n-p) nodes only. Hence the corresponding performance equation would be similar to (1) except that the coefficient matrix would be of order m now, i.e.,

$$\mathbf{I_{BUS} = Y_{BUS}^{new}\,E_{BUS}} \qquad \textbf{(2)}$$

Where Y_{BUS}^{new} is the bus admittance matrix of the reduced network and the vectors I_{BUS} and E_{BUS} are of order m. It is assumed in (1) that I_{BUS} and E_{BUS} are obtained with their elements arranged such that the elements associated with p nodes to be eliminated are in the lower portion of the vectors. Then the elements of Y_{BUS} also get located accordingly so that (1) after matrix partitioning yields,

$$\begin{array}{c} \\ \begin{vmatrix} \mathbf{I_{BUS\text{-}m}} \\ \mathbf{I_{BUS\text{-}p}} \end{vmatrix} \end{array} = \begin{array}{c c} & \begin{array}{cc} m & p \end{array} \\ \begin{array}{c} m \\ p \end{array} & \begin{vmatrix} \mathbf{Y_A} & \mathbf{Y_B} \\ \mathbf{Y_C} & \mathbf{Y_D} \end{vmatrix} \end{array} \begin{array}{c} \\ \begin{vmatrix} \mathbf{E_{BUS\text{-}m}} \\ \mathbf{E_{BUS\text{-}p}} \end{vmatrix} \end{array} \tag{3}$$

Where the self and mutual values of Y_A and Y_D are those identified only with the nodes to be retained and removed respectively and $Y_C=Y_B^t$ is composed of only the corresponding mutual admittance values, that are common to the nodes m and p.

Now, for the p nodes to be eliminated, it is necessary that, each element of the vector $I_{BUS\text{-}p}$ should be zero. Thus we have from (3):

$$I_{BUS\text{-}m} = Y_A\, E_{BUS\text{-}m} + Y_B\, E_{BUS\text{-}p}$$
$$I_{BUS\text{-}p} = Y_C\, E_{BUS\text{-}m} + Y_D\, E_{BUS\text{-}p} = 0 \tag{4}$$

Solving, $$\mathbf{E_{BUS\text{-}p} = - Y_D^{-1} Y_C\, E_{BUS\text{-}m}} \tag{5}$$

Thus, by simplification, we obtain an expression similar to (2) as,

$$\mathbf{I_{BUS\text{-}m}} = \{Y_A - Y_B Y_D^{-1} Y_C\}\, \mathbf{E_{BUS\text{-}m}} \tag{6}$$

Thus by comparing (2) and (6), we get an expression for the new bus admittance matrix in terms of the sub-matrices of the original bus admittance matrix as:

$$\mathbf{Y_{BUS}^{new} = \{Y_A - Y_B Y_D^{-1} Y_C\}} \tag{7}$$

This expression enables us to construct the given network with only the necessary nodes retained and all the unwanted nodes/buses eliminated. However, it can be observed from (7) that the expression involves finding the inverse of the sub-matrix Y_D (of order p). This would be computationally very tedious if p, the nodes to be eliminated is very large, especially for real practical systems. In such cases, it is more advantageous to eliminate the unwanted nodes from the given network by considering one node only at a time for elimination, as discussed next.

CASE-B: Separate Elimination of Nodes:

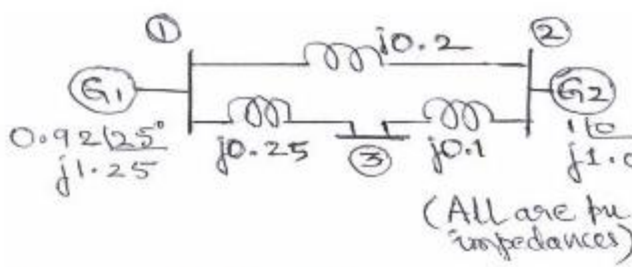
Here again, the system buses are to be renumbered, if necessary, such that the node to be removed always happens to be the last numbered one. The sub-matrix Y_D then would be a single element matrix and hence it inverse would be just equal to its own reciprocal value. Thus the generalized algorithmic equation for finding the elements of the new bus admittance matrix can be obtained from (6) as,

$$\mathbf{Y_{ij}}^{new} = \mathbf{Y_{ij}}^{old} - \mathbf{Y_{in}}\,\mathbf{Y_{nj}} / \mathbf{Y_{nn}} \qquad \forall\ i,j = 1,2,\ldots\ldots n. \tag{8}$$

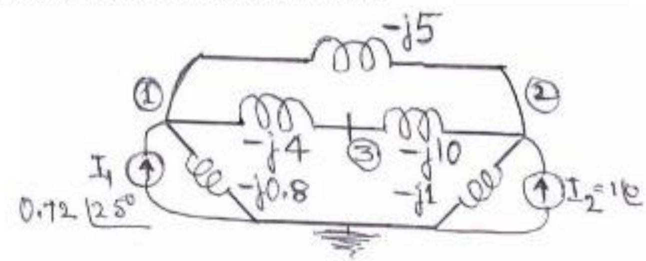
Each element of the original matrix must therefore be modified as per (7). Further, this procedure of eliminating the last numbered node from the given system of n nodes is to be iteratively repeated p times, so as to eliminate all the unnecessary p nodes from the original system.

Examples on Node elimination:

Example-1: *Obtain Y_{BUS} for the impedance network shown below by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*



The admittance equivalent network is as follows:



The bus admittance matrix is obtained by RoI as:

$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$

The reduced matrix after elimination of node 3 from the given system is determined as per the equation:

$$\mathbf{Y_{BUS}}^{\mathbf{New}} = \mathbf{Y_A - Y_B Y_D^{-1} Y_C}$$

$Y_{BUS}^{new} =$

n/n	1	2
1	-j8.66	j7.86
2	j7.86	-j8.66

Alternatively,

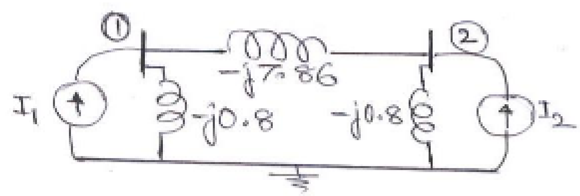
$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{i3}\, Y_{3j} / Y_{33}} \qquad \forall\ i,j = 1,2.$$

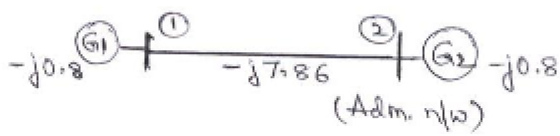
$$\mathbf{Y_{11} = Y_{11} - Y_{13}Y_{31}/ Y_{33} = -j8.66}$$

$$\mathbf{Y_{22} = Y_{22} - Y_{23}Y_{32}/ Y_{33} = -j8.66}$$

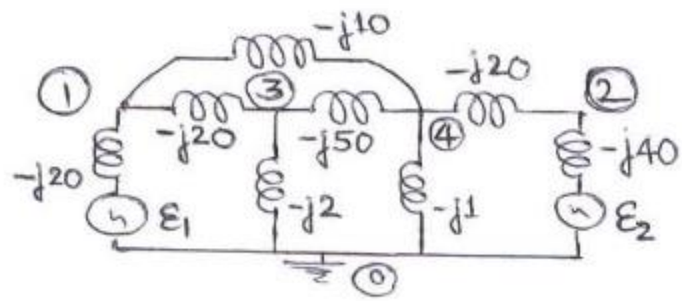
$$\mathbf{Y_{12} = Y_{21} = Y_{12} - Y_{13}Y_{32}/Y_{33} = j7.86}$$

Thus the reduced network can be obtained again by the rule of inspection as shown be low.





Example-2: *Obtain Y_{BUS} for the admittance network shown below by the rule of inspection. Also, determine Y_{BUS} for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.*

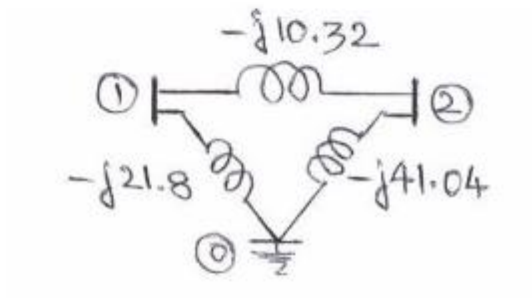


$$Y_{BUS} = \begin{array}{c|cccc} n/n & 1 & 2 & 3 & 4 \\ \hline 1 & -j50 & 0 & j20 & j10 \\ 2 & 0 & -j60 & 0 & j72 \\ 3 & j20 & 0 & -j72 & j50 \\ 4 & j10 & j72 & j50 & -j81 \end{array} = \begin{vmatrix} \mathbf{Y_A} & \mathbf{Y_B} \\ \mathbf{Y_C} & \mathbf{Y_D} \end{vmatrix}$$

$Y_{BUS}^{New} = \mathbf{Y_A - Y_B Y_D^{-1} Y_C}$

$$Y_{BUS}^{new} = \begin{array}{c|cc} n/n & 1 & 2 \\ \hline 1 & -j32.12 & j10.32 \\ 2 & j10.32 & -j51.36 \end{array}$$

Thus the reduced system of two nodes can be drawn by the rule of inspection as under:



Z_{BUS} building

FORMATION OF BUS IMPEDANCE MATRIX

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\overline{I}_{bus} \tag{9}$$

When expanded so as to refer to a n bus system, (9) will be of the form

$$\begin{aligned}
E_1 &= Z_{11}I_1 + Z_{12}I_2 + \dots\dots + Z_{1k}I_k \dots + Z_{1n}I_n \\
&\vdots \\
E_k &= Z_{k1}I_1 + Z_{k2}I_2 + \dots\dots + Z_{kk}I_k + \dots + Z_{kn}I_n \\
&\vdots \\
E_n &= Z_{n1}I_1 + Z_{n2}I_2 + \dots\dots + Z_{nk}I_k + \dots + Z_{nn}I_n
\end{aligned} \tag{10}$$

Now assume that the bus impedance matrix Z_{bus} is known for a partial network of m buses and a known reference bus. Thus, Z_{bus} of the partial network is of dimension $m \times m$. If now a new element is added between buses p and q we have the following two possibilities:

(i) p is an existing bus in the partial network and q is a new bus; in this case p-q is a **branch** added to the p-network as shown in Fig 1a, and

(ii) both p and q are buses existing in the partial network; in this case p-q is a **link** added to the p-network as shown in Fig 1b.

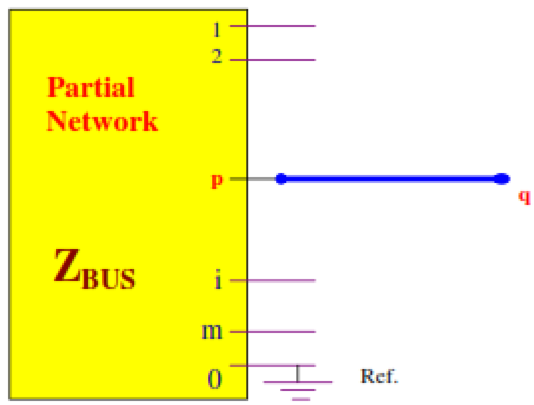


Fig 1a. Addition of branch p-q

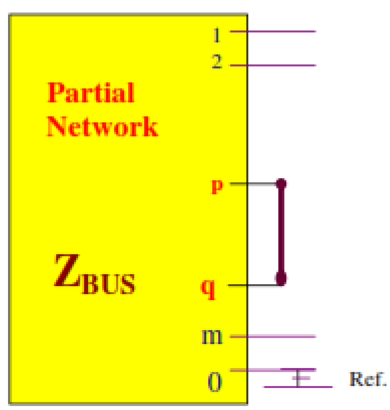


Fig 1b. Addition of link p-q

If the added element ia a branch, p-q, then the new bus impedance matrix would be of order m+1, and the analysis is confined to finding only the elements of the new row and column (corresponding to bus-q) introduced into the original matrix.

If the added element ia a link, p-q, then the new bus impedance matrix will remain unaltered with regard to its order. However, all the elements of the original matrix are updated to take account of the effect of the link added.

ADDITION OF A BRANCH

Consider now the performance equation of the network in impedance form with the added branch p-q, given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \tag{11}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector $y_{pq\text{-}rs}$ is not equal to zero and $Z_{ij} = Z_{ji} \quad \forall\, i,j = 1,2,\ldots m,q$ (12)

To find Z_{qi}:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

$E_k = Z_{ki} I_i = Z_{ki} \quad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots m, q$ (13)

Hence, $E_q = Z_{qi}$; $E_p = Z_{pi}$ ....

Also, $E_q = E_p - v_{pq}$; so that $Z_{qi} = Z_{pi} - v_{pq} \quad \forall\ i = 1, 2,\ldots i\ldots\ldots p,\ldots m, \neq q$ (14)

To find v_{pq}:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \tag{15}$$

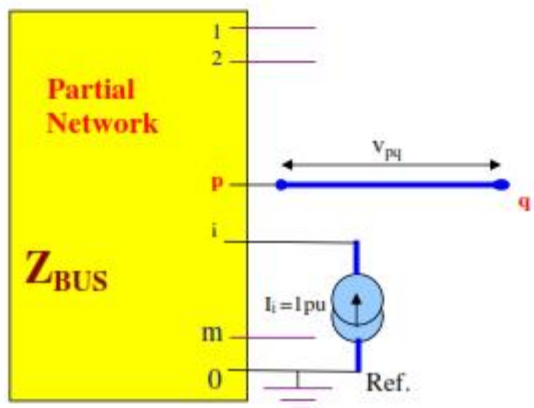


Fig.2 Calculation for Z_{qi}

where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \tag{16}$$

Solving, $v_{pq} = -\dfrac{\overline{y}_{pq,rs}\overline{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\overline{y}_{pq,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pq,pq}} \tag{17}$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\overline{y}_{pq,rs}\left(\overline{Z}_{ri} - \overline{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2......m; i \neq q \tag{18}$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu. As before, we have the relations as under:

$$E_k = Z_{kq} I_q = Z_{kq} \qquad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots m, q \tag{19}$$

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq} v_{pq} + \overline{y}_{pq,rs}\overline{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \dfrac{\overline{y}_{pq,rs}\overline{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\overline{y}_{pq,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \overline{y}_{pq,rs}\left(\overline{Z}_{rq} - \overline{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of branch to a p-network.

Case (a): If there is no mutual coupling then elements of $\overline{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

	$Z_{pi} = 0$	$i = 1,2......m : i \neq q$
And	$Z_{pq} = 0.$	
Hence, from (18) (22)	$Z_{qi} = 0$	$i = 1,2.......m; i \neq q$
And	$Z_{qq} = z_{pq,pq}$	\ (23)

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2 \ldots m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \tag{24}$$

ADDITION OF A LINK

Consider now the performance equation of the network in impedance form with the added link p-l, (p-l being a fictitious branch and l being a fictitious node) given by

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \tag{25}$$

It is assumed that the added branch *p-q* is mutually coupled with some elements of the partial network and since the network has bilateral passive elements only, we have

Vector $y_{pq\text{-}rs}$ is not equal to zero and $Z_{ij} = Z_{ji}$ $\quad \forall$ i,j=1,2,…m,l. (26)

<u>To find Z_{li}:</u>

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, e_l in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki}\, I_i = Z_{ki} \qquad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots m, l \tag{27}$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ………

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq}$ $\forall$ i=1,2,…i….p,...q,….m, ≠l (28)

To find v_{pq}:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \tag{29}$$

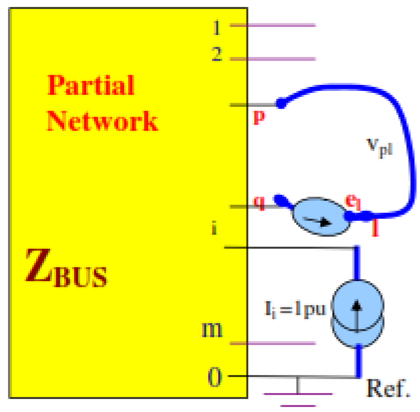


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl}v_{pl} + \overline{y}_{pl,rs}\overline{v}_{rs} = 0 \tag{30}$$

Solving, $v_{pl} = -\dfrac{\overline{y}_{pl,rs}\overline{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\overline{y}_{pl,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pl,pl}} \tag{31}$$

However,

$$\overline{y}_{pl,rs} = \overline{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \tag{32}$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\overline{y}_{pq,rs}\left(\overline{Z}_{ri} - \overline{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2.....m; i \neq l \tag{33}$$

<u>To find Z_{ll}:</u>

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu. As before, we have the relations as under:

$$E_k = Z_{kl}\, I_l = Z_{kl} \qquad \forall\ k = 1, 2,\ldots i\ldots\ldots p,\ldots q,\ldots.m, l \tag{34}$$

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

So that $Z_{ll} = Z_{pl} - Z_{ql} - v_{pl}$ $\forall$ i=1,2,…i….p,...q,….m, $\neq$l (35)

Since now the current in the added element is $i_{pl} = -I_l = -1.0$, we have from (29)

$$i_{pl} = y_{pl,pl}v_{pl} + \overline{y}_{pl,rs}\overline{v}_{rs} = -1$$

Solving, $v_{pl} = -1 + \dfrac{\overline{y}_{pl,rs}\overline{v}_{rs}}{y_{pl,pl}}$

$$v_{pl} = -1 + \frac{\overline{y}_{pl,rs}\left(\overline{E}_r - \overline{E}_s\right)}{y_{pl,pl}} \tag{36}$$

However,

$$\overline{y}_{pl,rs} = \overline{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \tag{37}$$

Using (34), (36) and (37) in (35), we get

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \tag{38}$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then E_p=0. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2....m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \tag{39}$$

From (39), it is thus observed that, when a link is added to a ref. bus, then the situation is similar to adding a branch to a fictitious bus and hence the following steps are followed:

1. The element is added similar to addition of a branch (case-b) to obtain the new matrix of order m+1.
2. The extra fictitious node, l is eliminated using the node elimination algorithm.

Case (d): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is not the reference node, then

$Z_{li} = Z_{pi} - Z_{qi}$

$$\begin{aligned} Z_{ll} &= Z_{pl} - Z_{ql} - z_{pq,pq} \\ &= Z_{pp} + Z_{qq} - 2\,Z_{pq} + z_{pq,pq} \end{aligned} \tag{40}$$

MODIFICATION OF Z_{BUS} FOR NETWORK CHANGES

An element which is not coupled to any other element can be removed easily. The Z_{bus} is modified as explained in sections above, by adding in parallel with the element (to be removed), a link whose impedance is equal to the negative of the impedance of the element to be removed. Similarly, the impedance value of an element which is not coupled to any other element can be changed easily. The Z_{bus} is modified again as explained in sections above, by adding in parallel with the element (whose impedance is to be changed), a link element of impedance value chosen such that the parallel equivalent impedance is equal to the desired value of impedance. When mutually coupled elements are removed, the Z_{bus} is modified by introducing appropriate changes in the bus currents of the original network to reflect the changes introduced due to the removal of the elements.

Examples on Z_{BUS} building

Example 1: *For the positive sequence network data shown in table below, obtain Z_{BUS} by building procedure.*

Sl. No.	p-q (nodes)	Pos. seq. reactance in pu
1	0-1	0.25
2	0-3	0.20
3	1-2	0.08
4	2-3	0.06

Solution:

The given network is as shown below with the data marked on it. Assume the elements to be added as per the given sequence: 0-1, 0-3, 1-2, and 2-3.

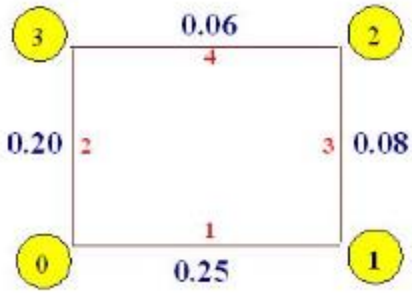
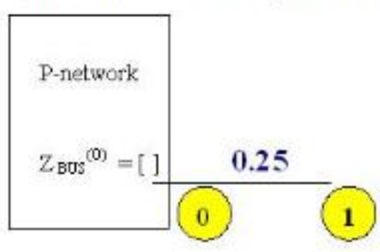


Fig. E1: Example System

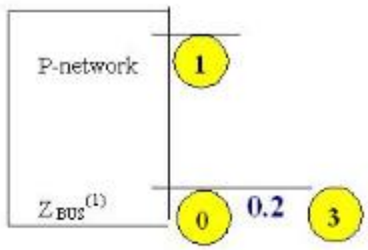
Consider building Z_{BUS} as per the various stages of building through the consideration of the corresponding partial networks as under:

Step-1: Add element–1 of impedance 0.25 pu from the external node-1 (q=1) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



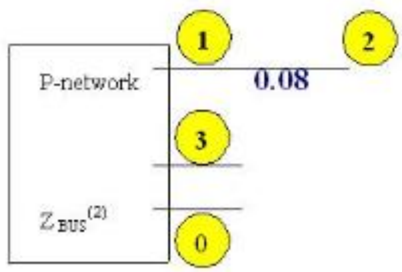
$$Z_{BUS}^{(1)} = \begin{array}{cc} & 1 \\ 1 & \begin{bmatrix} 0.25 \end{bmatrix} \end{array}$$

Step-2: Add element–2 of impedance 0.2 pu from the external node-3 (q=3) to internal ref. node-0 (p=0). (Case-a), as shown in the partial network;



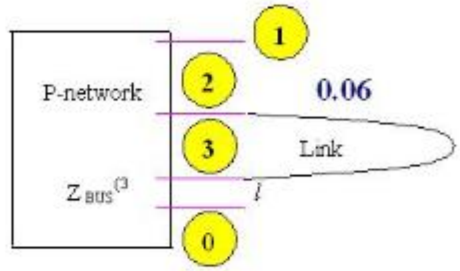
$$Z_{BUS}^{(2)} = \begin{array}{c|cc} & 1 & 3 \\ \hline 1 & 0.25 & 0 \\ 3 & 0 & 0.2 \end{array}$$

Step-3: Add element–3 of impedance 0.08 pu from the external node-2 (q=2) to internal node-1 (p=1). (Case-b), as shown in the partial network;



$$Z_{BUS}^{(3)} = \begin{array}{c|ccc} & 1 & 3 & 2 \\ \hline 1 & 0.25 & 0 & 0.25 \\ 3 & 0 & 0.2 & 0 \\ 2 & 0.25 & 0 & 0.33 \end{array}$$

Step-4: Add element–4 of impedance 0.06 pu between the two internal nodes, node-2 (p=2) to node-3 (q=3). (Case-d), as shown in the partial network;



$$Z_{BUS}^{(4)} = \begin{array}{c|cccc} & 1 & 3 & 2 & l \\ \hline 1 & 0.25 & 0 & 0.25 & 0.25 \\ 3 & 0 & 0.2 & 0 & -0.2 \\ 2 & 0.25 & 0 & 0.33 & 0.33 \\ l & 0.25 & -0.2 & 0.33 & 0.59 \end{array}$$

The fictitious node l is eliminated further to arrive at the final impedance matrix as under:

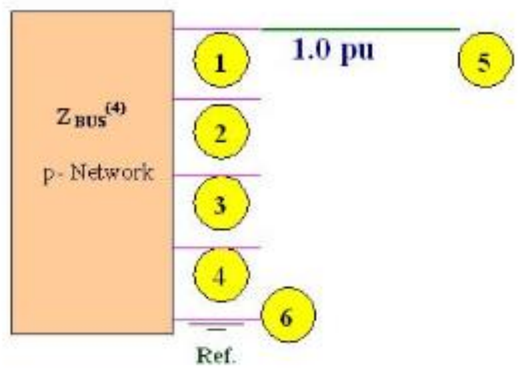
$$Z_{BUS}^{(final)} = \begin{array}{c|ccc} & 1 & 3 & 2 \\ \hline 1 & 0.1441 & 0.0847 & 0.1100 \\ 3 & 0.0847 & 0.1322 & 0.1120 \\ 2 & 0.1100 & 0.1120 & 0.1454 \end{array}$$

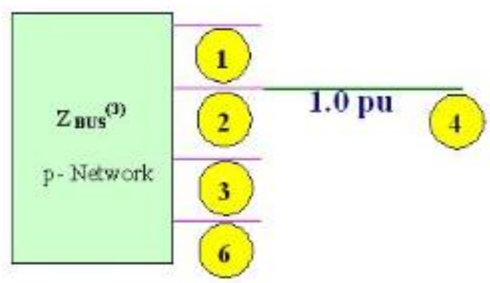
Example 2: *The Z_{BUS} for a 6-node network with bus-6 as ref. is as given below. Assuming the values as pu reactances, find the topology of the network and the parameter values of the elements involved. Assume that there is no mutual coupling of any pair of elements.*

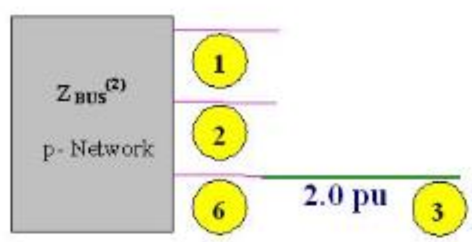
$$Z_{BUS} = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 2 & 0 & 2 & 0 \\ 3 & 0 & 0 & 2 & 0 & 0 \\ 4 & 0 & 2 & 0 & 3 & 0 \\ 5 & 2 & 0 & 0 & 0 & 3 \end{array}$$

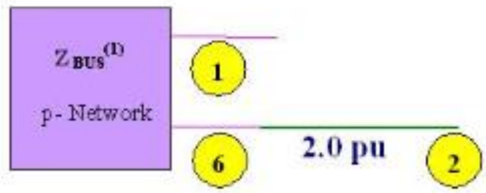
Solution:

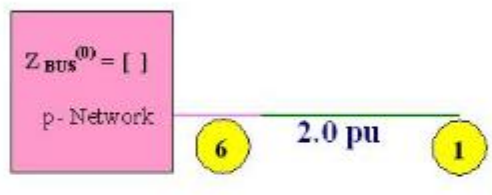
The specified matrix is so structured that by its inspection, we can obtain the network by backward analysis through the various stages of Z_{BUS} building and p-networks as under:


Z BUS(4)
p- Network
1
2
3
4
6
Ref.
1.0 pu
5


Z BUS(3)
p- Network
1
2
3
6
1.0 pu
4


Z BUS(2)
p- Network
1
2
6
2.0 pu
3





Thus the final network is with 6 nodes and 5 elements connected as follows with the impedance values of elements as indicated.

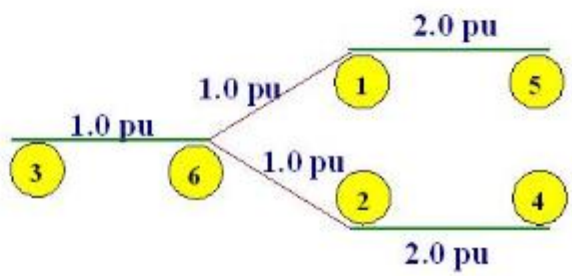
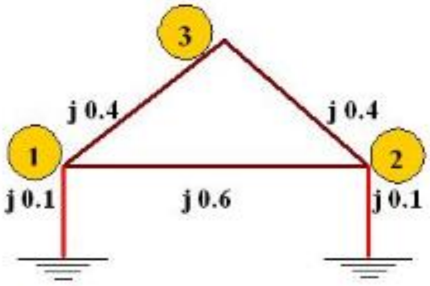


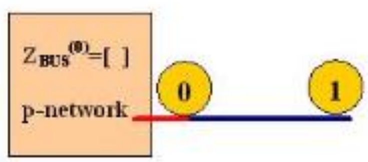
Fig. E2: Resultant network of example-2

Example 3: *Construct the bus impedance matrix for the system shown in the figure below by building procedure. Show the partial networks at each stage of building the matrix. Hence arrive at the bus admittance matrix of the system. How can this result be verified in practice?*



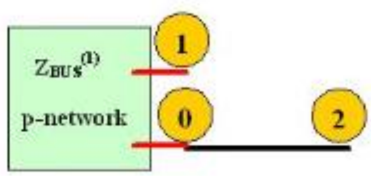
Solution: The specified system is considered with the reference node denoted by node-0. By its inspection, we can obtain the bus impedance matrix by building procedure by following the steps through the p-networks as under:

Step1: Add branch 1 between node 1 and reference node. (q =1, p = 0)



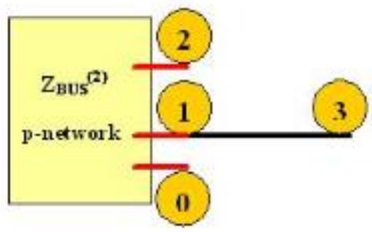
$$Z_{bus}^{(1)} = \begin{array}{c c} & 1 \\ 1 & \begin{bmatrix} j0.1 \end{bmatrix} \end{array}$$

Step2: Add branch 2, between node 2 and reference node. (q = 2, p = 0).



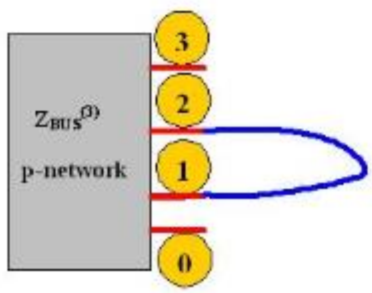
$$Z_{Bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} j0.1 & 0 \\ 0 & j0.15 \end{bmatrix} \end{array}$$

Step3: Add branch 3, between node 1 and node 3 (p = 1, q = 3)



$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j0.1 & 0 & j0.1 \\ 0 & j0.15 & 0 \\ j0.1 & 0 & j0.5 \end{bmatrix} \end{array}$$

Step 4: Add element 4, which is a link between node 1 and node 2. (p = 1, q = 2)



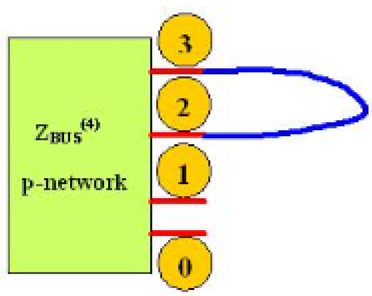
$$
\begin{array}{cc}
 & \begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ l \end{array} & Z_{bus} = \begin{bmatrix} j0.1 & 0 & j0.1 & j0.1 \\ 0 & j0.15 & 0 & -j0.15 \\ j0.1 & 0 & j0.5 & j0.1 \\ j0.1 & -j0.15 & j0.1 & j0.85 \end{bmatrix}
\end{array}
$$

Now the extra node-l has to be eliminated to obtain the new matrix of step-4, using the algorithmic relation:

$$\mathbf{Y_{ij}^{new} = Y_{ij}^{old} - Y_{in}\, Y_{nj} / Y_{nn}} \qquad \forall\ \mathbf{i}, j = 1, 2, 3.$$

$$
\begin{array}{cc}
 & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\
Z_{bus} = & \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 \\ j0.01765 & j0.12353 & j0.01765 \\ j0.08823 & j0.01765 & j0.48823 \end{bmatrix}
\end{array}
$$

Step 5: Add link between node 2 and node 3 (p = 2, q=3)



$$Z_{l1} = Z_{21} - Z_{31} = j0.01765 - j0.08823 = -j0.07058$$
$$Z_{l2} = Z_{22} - Z_{32} = j0.12353 - j0.01765 = j0.10588$$
$$Z_{l3} = Z_{23} - Z_{33} = j0.01765 - j0.48823 = -j0.47058$$
$$Z_{ll} = Z_{2l} - Z_{3l} + z_{23,23}$$
$$= j0.10588 - (-j0.47058) + j0.4 = j0.97646$$

Thus, the new matrix is as under:

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array} \begin{array}{c} \begin{array}{cccc} 1 & \quad 2 & \quad 3 & \quad l \end{array} \\ \begin{bmatrix} j0.08823 & j0.01765 & j0.08823 & -j0.07058 \\ j0.01765 & j0.12353 & j0.01765 & j0.10588 \\ j0.08823 & j0.01765 & j0.48823 & -j0.47058 \\ -j0.07058 & j0.10588 & -j0.47058 & j0.97646 \end{bmatrix} \end{array}$$

Node l is eliminated as shown in the previous step:

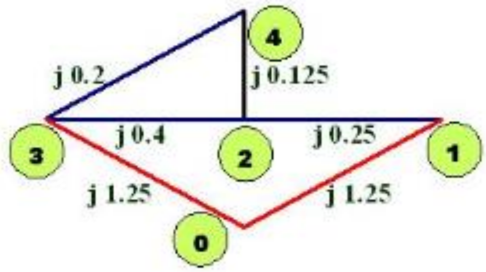
$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & \quad 2 & \quad 3 \end{array} \\ \begin{bmatrix} j0.08313 & j0.02530 & j0.05421 \\ j0.02530 & j0.11205 & j0.06868 \\ j0.05421 & j0.06868 & j0.26145 \end{bmatrix} \end{array}$$

Further, the bus admittance matrix can be obtained by inverting the bus impedance matrix as under:

$$Y_{bus} = [Z_{bus}]^{-1} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & \quad 2 & \quad 3 \end{array} \\ \begin{bmatrix} -j14.1667 & j1.6667 & j2.5 \\ j1.6667 & -j10.8334 & j2.5 \\ j2.5 & j2.5 & -j5.0 \end{bmatrix} \end{array}$$

As a check, it can be observed that the bus admittance matrix, Y_{BUS} can also be obtained by the rule of inspection to arrive at the same answer.

Example 4: Form the bus impedance matrix for the network shown below.



Solution:

Add the elements in the sequence, 0-1, 1-2, 2-3, 0-3, 3-4, 2-4, as per the various steps of building the matrix as under:

Step1: Add element 1, which is a branch between node-1 and reference node.

$$Z_{bus} = \begin{array}{c} \\ 1 \end{array}\begin{array}{c} 1 \\ \left[j1.25 \right] \end{array}$$

Step2: Add element 2, which is a branch between nodes 1 and 2.

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.5 \end{bmatrix} \end{array}$$

Step3: Add element 3, which is a branch between nodes 2 and 3

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} j1.25 & j1.25 & j1.25 \\ j1.25 & j1.5 & j1.5 \\ j1.25 & j1.5 & j1.9 \end{bmatrix} \end{array}$$

Step4: Add element 4, which is a link from node 3 to reference node.

$$
\begin{array}{c} \\ Z_{bus} = \end{array}
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ l \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & l \end{array} \\
\begin{bmatrix}
j1.25 & j1.25 & j1.25 & j1.25 \\
j1.25 & j1.5 & j1.5 & j1.5 \\
j1.25 & j1.5 & j1.9 & j1.9 \\
j1.25 & j1.5 & j1.9 & j3.15
\end{bmatrix}
\end{array}
$$

Eliminating node l,

$$
Z_{bus} = \begin{array}{c}
\begin{array}{ccc} 1 & 2 & 3 \end{array} \\
\begin{bmatrix}
j0.75397 & j0.65476 & j0.49603 \\
j0.65476 & j0.78571 & j0.59524 \\
j0.49603 & j0.59524 & j0.75397
\end{bmatrix}
\end{array}
$$

Step5: Add element 5, a branch between nodes 3 and 4.

$$
\begin{array}{c} \\ Z_{bus} = \end{array}
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix}
j0.75397 & j0.65476 & j0.49603 & j0.49603 \\
j0.65476 & j0.65476 & j0.59524 & j0.59524 \\
j0.49603 & j0.59524 & j0.75397 & j0.75397 \\
j0.49603 & j0.59524 & j0.75397 & j0.95397
\end{bmatrix}
\end{array}
$$

Step 6: Add element 6, a link between nodes 2 & 4.

$$
Z_{bus} = \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ l \end{array}
\begin{bmatrix}
j0.75397 & j0.65476 & j0.49603 & j0.49603 & j0.15873 \\
j0.65476 & j0.65476 & j0.59524 & j0.59524 & j0.19047 \\
j0.49603 & j0.59524 & j0.75397 & j0.75397 & -j0.15873 \\
j0.49603 & j0.59524 & j0.75397 & j0.95397 & -j0.35873 \\
j0.15873 & j0.19047 & -j0.15873 & -j0.35873 & j0.67421
\end{bmatrix}
$$

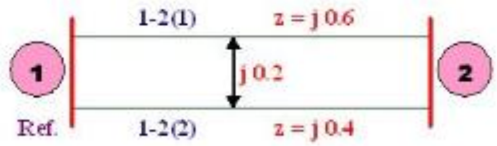
Eliminating node l we get the required bus impedance , matrix

$$Z_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} j0.7166 & j0.6099 & j0.5334 & j0.5805 \\ j0.6099 & j0.7319 & j0.6401 & j0.6966 \\ j0.5334 & j0.6401 & j0.7166 & j0.6695 \\ j0.5805 & j0.6966 & j0.6695 & j0.7631 \end{bmatrix} \end{array}$$

Example 5: Form the bus impedance matrix for the network data given below.

Element	Self Impedance		Mutual Impedance	
	Bus p-q	$z_{pq,pq}$ (pu)	Bus r-s	$z_{pq,rs}$ (pu)
1	1 – 2(1)	j0.6		
2	1 – 2(2)	j0.4	1 – 2(1)	j0.2

Solution:
Let bus-1 be the reference. Add the elements in the sequence 1-2(1), 1-2(2). Here, in the step-2, there is mutual coupling between the pair of elements involved.



Step1: Add element 1 from bus 1 to 2, element 1-2(1). (p=1, q=2, p is the reference node)

$$Z_{bus} = \begin{array}{c} \\ 2 \end{array}\begin{array}{c} 2 \\ [j0.6] \end{array}$$

Step2: Add element 2, element 1-2(2), which is a link from bus1 to 2, mutually coupled with element 1, 1-2(1).

$$Z_{bus} = \begin{array}{c} \\ 2 \\ l \end{array}\begin{array}{c} \begin{array}{cc} 2 & l \end{array} \\ \begin{bmatrix} j0.6 & Z_{2l} \\ Z_{l2} & Z_{ll} \end{bmatrix} \end{array}$$

Where,

$$Z_{2l} = Z_{l2} = -Z_{22} + \frac{y_{12(2)12(1)}\left(Z_{12} - Z_{22}\right)}{y_{12(2)12(2)}}$$

$Z_{12} = Z_{1l} = 0$ (as bus 1 is reference)

Consider the primitive impedance matrix for the two elements given by

$$[z] = \begin{array}{c c} & \begin{array}{cc} 1\text{-}2(1) & 1\text{-}2(2) \end{array} \\ \begin{array}{c} 1-2(1) \\ 1-2(2) \end{array} & \begin{bmatrix} j0.6 & j0.2 \\ j0.2 & j0.4 \end{bmatrix} \end{array}$$

Thus the primitive admittance matrix is obtained by taking the inverse of [z] as

$$[y] = \begin{array}{c c} & \begin{array}{cc} 1\text{-}2(1) & 1\text{-}2(2) \end{array} \\ \begin{array}{c} 1-2(1) \\ 1-2(2) \end{array} & \begin{bmatrix} -j2.0 & j1.0 \\ j1.0 & -j3.0 \end{bmatrix} \end{array}$$

Thus,

$y_{12(1),\,12(2)} =$ j1.0; $\quad y_{12(2),\,12(2)} =$ -j3.0

So that we have,

$$Z_{2l} = Z_{l2} = -j0.6 + \frac{(j1.0)(-j0.6)}{-j3.0} = -j0.4$$

$$Z_{ll} = -Z_{2l} + \frac{1 + y_{12(2),12(1)}(Z_{1l} - Z_{2l})}{y_{12(2),12(2)}} = j0.4 + \frac{1 + (j1.0)(j0.4)}{-j3.0} = j0.6$$

$$Z_{bus} = \begin{array}{c c} & \begin{array}{cc} 2 & l \end{array} \\ \begin{array}{c} 2 \\ l \end{array} & \begin{bmatrix} j0.6 & -j0.4 \\ -j0.4 & j0.6 \end{bmatrix} \end{array}$$

Thus, the network matrix corresponding to the 2-node, 1-bus network given, is obtained after eliminating the extra node-l as a single element matrix, as under:

$$Z_{bus} = \begin{array}{c c} & 2 \\ 2 & [j0.3333] \end{array}$$

Power Flow Analysis

Introduction

- **A power flow study** (**load-flow study**) is a steady-state analysis whose target is to determine the voltages, currents, and real and reactive power flows in a system under a given load conditions.
- The purpose of power flow studies is to plan ahead and account for various hypothetical situations. For example, if a transmission line is be taken off line for maintenance, can the remaining lines in the system handle the required loads without exceeding their rated values.

Power-flow analysis equations

The basic equation for power-flow analysis is derived from the nodal analysis equations for the power system: For example, for a 4-bus system,

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

where Y_{ij} are the elements of the bus admittance matrix, V_i are the bus voltages, and I_i are the currents injected at each node. The node equation at bus *i* can be written as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j$$

Power-flow analysis equations

Relationship between per-unit real and reactive power supplied to the system at bus i and the per-unit current injected into the system at that bus:

$$S_i = V_i I_i^* = P_i + jQ_i$$

where V_i is the per-unit voltage at the bus; I_i^* - complex conjugate of the per-unit current injected at the bus; P_i and Q_i are per-unit real and reactive powers. Therefore,

$$I_i^* = (P_i + jQ_i)/V_i \quad \Rightarrow I_i = (P_i - jQ_i)/V_i^*$$

$$\Rightarrow P_i - jQ_i = V_i^* \sum_{j=1}^{n} Y_{ij} V_j = \sum_{j=1}^{n} Y_{ij} V_j V_i^*$$

Power flow equations

Let $$Y_{ij} = |Y_{ij}| \angle \theta_{ij} \quad and \quad V_i = |V_i| \angle \delta_i$$

Then $$P_i - jQ_i = \sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \angle(\theta_{ij} + \delta_j - \delta_i)$$

Hence, $$P_i = \sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \cos(\theta_{ij} + \delta_j - \delta_i)$$

and $$Q_i = -\sum_{j=1}^{n} |Y_{ij}||V_j||V_i| \sin(\theta_{ij} + \delta_j - \delta_i)$$

Formulation of power-flow study

- There are 4 variables that are associated with each bus:
  - P,
  - Q,
  - V,
  - δ.
- Meanwhile, there are two power flow equations associated with each bus.
- In a power flow study, two of the four variables are defined an the other two are unknown. That way, we have the same number of equations as the number of unknown.
- The known and unknown variables depend on the type of bus.

Formulation of power-flow study

Each bus in a power system can be classified as one of three types:

1. **Load bus (P-Q bus)** – a buss at which the real and reactive power are specified, and for which the bus voltage will be calculated. All busses having no generators are load busses. In here, **V and δ are unknown**.

2. **Generator bus (P-V bus)** – a bus at which the magnitude of the voltage is defined and is kept constant by adjusting the field current of a synchronous generator. We also assign real power generation for each generator according to the economic dispatch. In here, **Q and δ are unknown**

3. **Slack bus (swing bus)** – a special generator bus serving as the reference bus. Its voltage is assumed to be fixed in both magnitude and phase (for instance, $1\angle 0^{\circ}$ pu). In here, **P and Q are unknown**.

Formulation of power-flow study

- Note that the power flow equations are non-linear, thus cannot be solved analytically. A numerical iterative algorithm is required to solve such equations. A standard procedure follows:
  1. Create a bus admittance matrix Y_{bus} for the power system;
  2. Make an initial estimate for the voltages (both magnitude and phase angle) at each bus in the system;
  3. Substitute in the power flow equations and determine the deviations from the solution.
  4. Update the estimated voltages based on some commonly known numerical algorithms (e.g., Newton-Raphson or Gauss-Seidel).
  5. Repeat the above process until the deviations from the solution are minimal.

Example

Consider a 4-bus power system below. Assume that

- bus 1 is the slack bus and that it has a voltage $V1 = 1.0\angle 0°$ pu.
- The generator at bus 3 is supplying a real power $P3 = 0.3$ pu to the system with a voltage magnitude 1 pu.
- The per-unit real and reactive power loads at busses 2 and 4 are $P2 = 0.3$ pu, $Q2 = 0.2$ pu, $P4 = 0.2$ pu, $Q4 = 0.15$ pu.

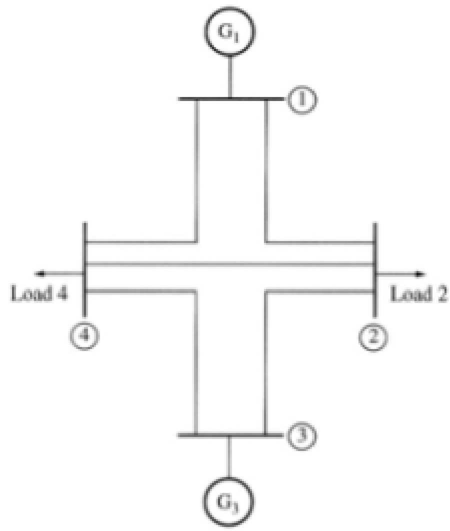


Table of Buses:

Bus 1	Slack bus
Bus 2	Load bus
Bus 3	Generator bus
Bus 4	Load bus

Example (cont.)

- Y-bus matrix (refer to example in book)

$$Y_{bus} = \begin{bmatrix} 1.7647 - j7.0588 & -0.5882 + j2.3529 & 0 & -1.1765 + j4.7059 \\ -0.5882 + j2.3529 & 1.5611 - j6.6290 & -0.3846 + j1.9231 & -0.5882 + j2.3529 \\ 0 & -0.3846 + j1.9231 & 1.5611 - j6.6290 & -1.1765 + j4.7059 \\ -1.1765 + j4.7059 & -0.5882 + j2.3529 & -1.1765 + j4.7059 & 2.9412 - j11.7647 \end{bmatrix}$$

- Power flow solution:

$$\begin{aligned} V_1 &= 1.0\angle 0^\circ \ pu \\ V_2 &= 0.964\angle -0.97^\circ \ pu \\ V_3 &= 1.0\angle 1.84^\circ \ pu \\ V_4 &= 0.98\angle -0.27^\circ \ pu \end{aligned}$$

- By knowing the node voltages, the power flow (both active and reactive) in each branch of the circuit can easily be calculated.

CHAPTER 3

LOAD FLOW ANALYSIS

[CONTENTS: *Review of solution of equations, direct and iterative methods, classification of buses, importance of slack bus and Y_{BUS} based analysis, constraints involved, load flow equations, GS method: algorithms for finding the unknowns, concept of acceleration of convergence, NR method- algorithms for finding the unknowns, tap changing transformers, Fast decoupled load flow, illustrative examples*]

REVIEW OF NUMERICAL SOLUTION OF EQUATIONS

The numerical analysis involving the solution of algebraic simultaneous equations forms the basis for solution of the performance equations in computer aided electrical power system analyses, such as during linear graph analysis, load flow analysis (nonlinear equations), transient stability studies (differential equations), etc. Hence, it is necessary to review the general forms of the various solution methods with respect to all forms of equations, as under:

1. **Solution Linear equations**:

   * **Direct methods**:

     - Cramer's (Determinant) Method,
     - Gauss Elimination Method (only for smaller systems),
     - LU Factorization (more preferred method), etc.

   * **Iterative methods**:

     - Gauss Method
     - Gauss-Siedel Method (for diagonally dominant systems)

2. **Solution of Nonlinear equations:**

   Iterative methods only:

     - Gauss-Siedel Method (for smaller systems)
     - Newton-Raphson Method (if corrections for variables are small)

3. **Solution of differential equations:**

   Iterative methods only:

     - Euler and Modified Euler method,
     - RK IV-order method,
     - Milne's predictor-corrector method, etc.

It is to be observed that the nonlinear and differential equations can be solved only by the iterative methods. The iterative methods are characterized by the various performance features as under:

- Selection of initial solution/ estimates
- Determination of fresh/ new estimates during each iteration
- Selection of number of iterations as per tolerance limit
- Time per iteration and total time of solution as per the solution method selected
- Convergence and divergence criteria of the iterative solution
- Choice of the Acceleration factor of convergence, etc.

A comparison of the above solution methods is as under:

- In general, the direct methods yield exact or accurate solutions. However, they are suited for only the smaller systems, since otherwise, in large systems, the possible round-off errors make the solution process inaccurate.
- The iterative methods are more useful when the diagonal elements of the coefficient matrix are large in comparison with the off diagonal elements. The round-off errors in these methods are corrected at the successive steps of the iterative process.
- The Newton-Raphson method is very much useful for solution of non –linear equations, if all the values of the corrections for the unknowns are very small in magnitude and the initial values of unknowns are selected to be reasonably closer to the exact solution.

LOAD FLOW STUDIES

Introduction: Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition. Different steady state solutions can be obtained, for different operating conditions, to help in planning, design and operation of the power system.

Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known. Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc. Load flow studies are required for deciding the economic operation of the power system. They are also required in transient stability studies. Hence, load-flow studies play a vital role in power system studies.

Thus the load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions. At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle. Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only. The constraints placed on the load flow solutions could be:

- The Kirchhoff's relations holding good,
- Capability limits of reactive power sources,
- Tap-setting range of tap-changing transformers,
- Specified power interchange between interconnected systems,
- Selection of initial values, acceleration factor, convergence limit, etc.

Classification of buses for LFA: Different types of buses are present based on the specified and unspecified variables at a given bus as presented in the table below:

Table 1. Classification of buses for LFA

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G,Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

Importance of swing bus: The slack or swing bus is usually a PV-bus with the largest capacity generator of the given system connected to it. The generator at the swing bus supplies the power difference between the "specified power into the system at the other buses" and the "total system output plus losses". Thus swing bus is needed to supply the additional real and reactive power to meet the losses. Both the magnitude and phase angle of voltage are specified at the swing bus, or otherwise, they are assumed to be equal to 1.0 p.u. and 0^0 , as per flat-start procedure of iterative solutions. The real and reactive powers at the swing bus are found by the computer routine as part of the load flow solution process. It is to be noted that the source at the swing bus is a perfect one, called the swing machine, or slack machine. It is voltage regulated, i.e., the magnitude of voltage fixed. The phase angle is the system reference phase and hence is fixed. The generator at the swing bus has a torque angle and excitation which vary or swing as the demand changes. This variation is such as to produce fixed voltage.

Importance of Y_{BUS} based LFA: The majority of load flow programs employ methods using the bus admittance matrix, as this method is found to be more economical. The bus admittance matrix plays a very important role in load floe analysis. It is a complex, square and symmetric matrix and hence only n(n+1)/2 elements of Y_{BUS} need to be stored for a n-bus system. Further, in the Y_{BUS} matrix, Y_{ij} = 0, if an incident element is not present in the system connecting the buses 'i' and 'j'. since in a large power system, each bus is connected only to a fewer buses through an incident element, (about 6-8), the coefficient matrix, Y_{BUS} of such systems would be highly sparse, i.e., it will have many zero valued elements in it. This is defined by the sparsity of the matrix, as under:

$$\text{Percentage sparsity of a given matrix of } n^{th} \text{ order:} = \frac{\text{Total no. of zero valued elements of } Y_{BUS}}{\text{Total no. of entries of } Y_{BUS}}$$

$$\mathbf{S} = (\mathbf{Z} / \mathbf{n}^2) \times \mathbf{100}\ \% \qquad \mathbf{(1)}$$

The percentage sparsity of Y_{BUS}, in practice, could be as high as 80-90%, especially for very large, practical power systems. This sparsity feature of Y_{BUS} is extensively used in reducing the load flow calculations and in minimizing the memory required to store the coefficient matrices. This is due to the fact that only the non-zero elements Y_{BUS} can be stored during the computer based implementation of the schemes, by adopting the suitable optimal storage schemes. While Y_{BUS} is thus highly sparse, it's inverse, Z_{BUS}, the bus impedance matrix is not so. It is a FULL matrix, unless the optimal bus ordering schemes are followed before proceeding for load flow analysis.

THE LOAD FLOW PROBLEM

Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities. The first step in the analysis is the formulation of suitable equations for the power flows in the system. The power system is a large interconnected system, where various buses are connected by transmission lines. At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present. The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power S_i (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \qquad (2)$$

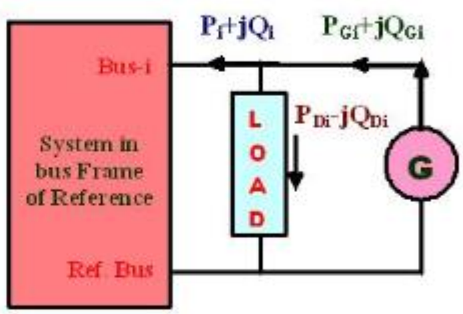


Fig.1 power flows at a bus-i

where S_i = net complex power injected into bus *i*, S_{Gi} = complex power injected by the generator at bus *i*, and S_{Di} = complex power drawn by the load at bus *i*. According to conservation of complex power, at any bus i, the complex power injected into the bus must be equal to the sum of complex power flows out of the bus via the transmission lines. Hence,

$$S_i = \sum S_{ij} \qquad \forall\, i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(3)}$$

where S_{ij} is the sum over all lines connected to the bus and n is the number of buses in the system (excluding the ground). The bus current injected at the bus-i is defined as

$$I_i = I_{Gi} - I_{Di} \qquad \forall\, i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(4)}$$

where I_{Gi} is the current injected by the generator at the bus and I_{Di} is the current drawn by the load (demand) at that bus. In the bus frame of reference

$$I_{BUS} = Y_{BUS}\, V_{BUS} \qquad \textbf{(5)}$$

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\, i = 1, 2, \ldots\ldots\ldots n \qquad \textbf{(6)}$$

The complex power S_i is given by

$$\begin{aligned} S_i &= V_i\, I_i^* \\ &= V_i \left(\sum_{j=1}^{n} Y_{ij}\, V_j \right)^* \\ &= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \end{aligned} \qquad \textbf{(7)}$$

Let $\quad V_i \underline{\Delta} |V_i| \angle \delta_i = |V_i| (\cos \delta_i + j \sin \delta_i)$

$\delta_{ij} = \delta_i - \delta_j$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(\cos\delta_{ij} + j\sin\delta_{ij}\right)\left(G_{ij} - jB_{ij}\right) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(G_{ij}\cos\delta_{ij} + B_{ij}\sin\delta_{ij}\right) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,\left(G_{ij}\sin\delta_{ij} - B_{ij}\cos\delta_{ij}\right) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\qquad Y_{ij} = |Y_{ij}|\,\angle\theta_{ij}$ $\qquad$ **(11)**

Again, we get from (7),

$$S_i = |V_i|\angle\delta_i \sum_{j=1}^{n} |Y_{ij}|\,\angle -\theta_{ij}\,|V_j|\,\angle -\delta_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos\left(-\theta_{ij} + \delta_i - \delta_j\right)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\cos -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\, i = 1, 2, \ldots\ldots\ldots\ldots n, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\,|V_j|\,\sin -(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i|\,|V_j|\,|Y_{ij}|\,\sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\, i = 1, 2, \ldots\ldots\ldots\ldots n \qquad \textbf{(14)}$$

Equations (9)-(10) and (13)-(14) are the 'power flow equations' or the 'load flow equations' in two alternative forms, corresponding to the n-bus system, where each bus-*i* is characterized by four variables, P_i, Q_i, $|V_i|$, and δ_i. Thus a total of 4n variables are involved in these equations. The load flow equations can be solved for any 2n unknowns, if the other 2n variables are specified. This establishes the need for classification of buses of the system for load flow analysis into: PV bus, PQ bus, etc.

DATA FOR LOAD FLOW

Irrespective of the method used for the solution, the data required is common for any load flow. All data is normally in pu. The bus admittance matrix is formulated from these data. The various data required are as under:

System data: It includes: number of buses-*n*, number of PV buses, number of loads, number of transmission lines, number of transformers, number of shunt elements, the slack bus number, voltage magnitude of slack bus (angle is generally taken as 0^0), tolerance limit, base MVA, and maximum permissible number of iterations.

Generator bus data: For every PV bus *i*, the data required includes the bus number, active power generation P_{Gi}, the specified voltage magnitude $|V_{i,sp}|$, minimum reactive power limit $Q_{i,min}$, and maximum reactive power limit $Q_{i,max}$.

Load data: For all loads the data required includes the the bus number, active power demand P_{Di}, and the reactive power demand Q_{Di}.

Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the starting bus number *i*, ending bus number *k*,.resistance of the line, reactance of the line and the half line charging admittance.

Transformer data:

For every transformer connected between buses *i* and *k* the data to be given includes: the starting bus number *i*, ending bus number *k*, resistance of the transformer, reactance of the transformer, and the off nominal turns-ratio *a*.

Shunt element data: The data needed for the shunt element includes the bus number where element is connected, and the shunt admittance (G_{sh} + j B_{sh}).

GAUSS – SEIDEL (GS) METHOD

The GS method is an iterative algorithm for solving non linear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached. The GS method applied to power flow problem is as discussed below.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad \textbf{(15)}$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad \textbf{(16)}$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \qquad \forall\, i = 2, 3, \ldots\ldots\ldots\ldots n \qquad \textbf{(17)}$$

Equation (17) is an implicit equation since the unknown variable, appears on both sides of the equation. Hence, it needs to be solved by an iterative technique. Starting from an initial estimate of all bus voltages, in the RHS of (17) the most recent values of the bus voltages is substituted. One iteration of the method involves computation of all the bus voltages. In Gauss–Seidel method, the value of the updated voltages are used in the computation of subsequent voltages in the same iteration, thus speeding up convergence. Iterations are carried out till the magnitudes of all bus voltages do not change by more than the tolerance value. Thus the algorithm for GS method is as under:

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix Y_{BUS}. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be $1.0\angle 0^0$. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any $(k+1)^{st}$ iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \forall \text{ i=2,3,...n} \qquad \textbf{(18)}$$

   Here note that when computation is carried out for bus-i, updated values are already available for buses 2,3….(i-1) in the current $(k+1)^{st}$ iteration. Hence these values are used. For buses $(i+1)$.....n, values from previous, k^{th} iteration are used.

5. Continue iterations till

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall \text{ i = 2,3,...n} \qquad \textbf{(19)}$$

   Where, ε is the tolerance value. Generally it is customary to use a value of 0.0001 pu.

6. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad \textbf{(20)}$$

7. Compute all line flows.
8. The complex power loss in the line is given by $S_{ik} + S_{ki}$. The total loss in the system is calculated by summing the loss over all the lines.

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Q_i to be used in (18). From (15) we have

$$Q_i = -\operatorname{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij} V_j\right\}$$

Where *Im* stands for the imaginary part. At any $(k+1)^{st}$ iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\operatorname{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij} V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \qquad \textbf{(22)}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $Q_i^{(k+1)}$ computed using (21) is either less than $Q_{i,\,min}$ or greater than $Q_{i,max}$, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the $(k+1)^{st}$ iteration and the voltage is calculated with the value of Q_i set as follows:

If $Q_i < Q_{i,min}$	If $Q_i > Q_{i,max}$
Then $Q_i = Q_{i,min.}$	Then $Q_i = Q_{i,max.}$

(23)

If in the subsequent iteration, if Q_i falls within the limits, then the bus can be switched back to PV status.

<u>Acceleration of convergence</u>

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerated) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \quad \textbf{(24)}$$

where α is a real number. When α = 1, the value of $V_i^{(k+1)}$ is the computed value. If $1 < \alpha < 2$, then the value computed is extrapolated. Generally α is taken between 1.2 to 1.6, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

Examples on GS load flow analysis:

Example-1: *Obtain the voltage at bus 2 for the simple system shown in Fig 2, using the Gauss–Seidel method, if $V_1 = 1\angle 0^0$ pu.*

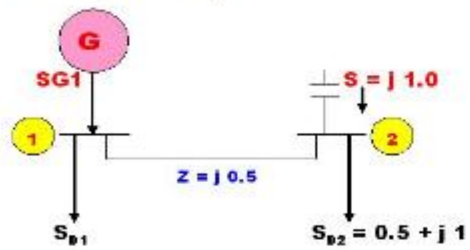


Fig : System of Example 1

Solution:

Here the capacitor at bus 2, injects a reactive power of 1.0 pu. The complex power injection at bus 2 is

$$S_2 = j1.0 - (0.5 + j\,1.0) = -\,0.5 \text{ pu.}$$

$$V_1 = 1\angle 0^0$$

$$Y_{BUS} = \begin{bmatrix} -j2 & j2 \\ j2 & -j2 \end{bmatrix}$$

$$V_2^{(k+1)} = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{\left(V_2^{(k)}\right)^*} - Y_{21}\,V_1\right]$$

Since V_1 is specified it is a constant through all the iterations. Let the initial voltage at bus 2, $V_2^{o} = 1 + j\,0.0 = 1\angle 0^0$ pu.

$$V_2^1 = \frac{1}{-j2}\left[\frac{-0.5}{1\angle 0^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 1.0 - j0.25 = 1.030776\ \angle -14.036^0$$

$$V_2^2 = \frac{1}{-j2}\left[\frac{-0.5}{1.030776\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.94118 - j\ 0.23529\ = 0.970145\ \angle -14.036^0$$

$$V_2^3 = \frac{1}{-j2}\left[\frac{-0.5}{0.970145\angle 14.036^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.9375 - j\ 0.249999\ = 0.970261\ \angle -14.931^0$$

$$V_2^4 = \frac{1}{-j2}\left[\frac{-0.5}{0.970261\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933612 - j\ 0.248963 = 0.966237\ \angle -14.931^0$$

$$V_2^5 = \frac{1}{-j2}\left[\frac{-0.5}{0.966237\angle 14.931^0} - \left(j2\times 1\angle 0^0\right)\right]$$

$$= 0.933335 - j\ 0.25\ \ = 0.966237\ \angle -14.995^0$$

Since the difference in the voltage magnitudes is less than 10^{-6} pu, the iterations can be stopped. To compute line flow

$$I_{12} = \frac{V_1 - V_2}{Z_{12}} = \frac{1\angle 0^0\ - 0.966237\angle -14.995^0}{j0.5}$$

$$= 0.517472\ \angle -14.931^0$$

$$S_{12} = V_1 I_{12}^* = 1\angle 0^0 \times 0.517472\ \angle\ 14.931^0$$

$$= 0.5 + j\ 0.133329\ \text{pu}$$

$$I_{21} = \frac{V_2 - V_1}{Z_{12}} = \frac{0.966237\angle -14.995^0 - 1\angle 0^0}{j0.5}$$

$$= 0.517472\ \angle -194.93^0$$

$$S_{21} = V_2 I_{21}^* = -\ 0.5 + j\ 0.0\ \text{pu}$$

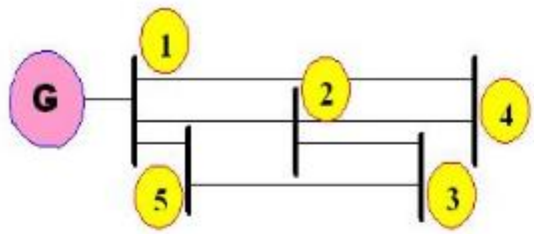
The total loss in the line is given by

$$S_{12} + S_{21} =\ j\ 0.133329\ \text{pu}$$

Obviously, it is observed that there is no real power loss, since the line has no resistance.

Example-2:

For the power system shown in fig. below, with the data as given in tables below, obtain the bus voltages at the end of first iteration, by applying GS method.



Power System of Example 2

Line data of example 2

SB	EB	R (pu)	X (pu)	$\frac{B_C}{2}$
1	2	0.10	0.40	-
1	4	0.15	0.60	-
1	5	0.05	0.20	-
2	3	0.05	0.20	-
2	4	0.10	0.40	-
3	5	0.05	0.20	-

Bus data of example 2

Bus No.	P_G (pu)	Q_G (pu)	P_D (pu)	Q_D (pu)	$\lvert V_{sp}\rvert$ (pu)	δ
1	-	-	-	-	1.02	0^0
2	-	-	0.60	0.30	-	-
3	1.0	-	-	-	1.04	-
4	-	-	0.40	0.10	-	-
5	-	-	0.60	0.20	-	-

Solution: In this example, we have,

- Bus 1 is slack bus, Bus 2, 4, 5 are PQ buses, and Bus 3 is PV bus
- The lines do not have half line charging admittances

  $P_2 + jQ_2 = P_{G2} + jQ_{G2} - (P_{D2} + jQ_{D2}) = -0.6 - j0.3$

$P_3 + jQ_3 = P_{G3} + jQ_{G3} - (P_{D3} + jQ_{D3}) = 1.0 + jQ_{G3}$

Similarly $P_4 + jQ_4 = -0.4 - j0.1, \quad P_5 + jQ_5 = -0.6 - j0.2$

The Y_{bus} formed by the rule of inspection is given by:

$Y_{bus} =$

2.15685 -j8.62744	-0.58823 +j2.35294	0.0+j0.0	-0.39215 +j1.56862	-1.17647 +j4.70588
-0.58823 +j2.35294	2.35293 -j9.41176	-1.17647 +j4.70588	-0.58823 +j2.35294	0.0+j0.0
0.0+j0.0	-1.17647 +j4.70588	2.35294 -j9.41176	0.0+j0.0	-1.17647 +j4.70588
-0.39215 +j1.56862	-0.58823 +j2.35294	0.0+j0.0	0.98038 -j3.92156	0.0+j0.0
-1.17647 +j4.70588	0.0+j0.0	-1.17647 +j4.70588	0.0+j0.0	2.35294 -j9.41176

The voltages at all PQ buses are assumed to be equal to 1+j0.0 pu. The slack bus voltage is taken to be $V_1^0 = 1.02+j0.0$ in all iterations.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{o*}} - Y_{21}V_1^o - Y_{23}V_3^0 - Y_{24}V_4^0 - Y_{25}V_5^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{-0.6 + j0.3}{1.0 - j0.0} - \{(-0.58823 + j2.35294) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 0^o\} - \{(-0.58823 + j2.35294) \times 1.0\angle 0^o\}\right]$$

$$= 0.98140\angle -3.0665^0 = 0.97999 - j0.0525$$

Bus 3 is a PV bus. Hence, we must first calculate Q_3. This can be done as under:

$$Q_3 = |V_3||V_1|(G_{31}\sin\delta_{31} - B_{31}\cos\delta_{31}) + |V_3||V_2|(G_{32}\sin\delta_{32} - B_{32}\cos\delta_{32})$$

$$+ |V_3|^2(G_{33}\sin\delta_{33} - B_{33}\cos\delta_{33}) + |V_3||V_4|(G_{34}\sin\delta_{34} - B_{34}\cos\delta_{34})$$

$$+ |V_3||V_5|(G_{35}\sin\delta_{35} - B_{35}\cos\delta_{35})$$

We note that $\delta_1 = 0^0$; $\delta_2 = -3.0665^0$; $\delta_3 = 0^0$; $\delta_4 = 0^0$ and $\delta_5 = 0^0$

$\therefore \delta_{31} = \delta_{33} = \delta_{34} = \delta_{35} = 0^0$ ($\delta_{ik} = \delta_i - \delta_k$); $\quad \delta_{32} = 3.0665^0$

$Q_3 = 1.04$ [1.02 (0.0+j0.0) + 0.9814 {$-1.17647 \times \sin(3.0665^0) - 4.70588$ $\times\cos(3.0665^0)$}+1.04{$-9.41176 \times\cos(0^0)$}+1.0 {0.0 + j0.0}+1.0{$-4.70588\times\cos(0^0)$}]

$= 1.04$ [$-4.6735 + 9.78823 - 4.70588$] $= 0.425204$ pu.

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}V_1^o - Y_{32}V_2^1 - Y_{34}V_4^0 - Y_{35}V_5^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{1.0 - j0.425204}{1.04 - j0.0} - \{(-1.7647 + j4.70588) \times (0.98140\angle -3.0665^o)\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times (1\angle 0^o)\}\right]$$

$$= 1.05569 \angle 3.077^0 = 1.0541 + j0.05666 \text{ pu.}$$

Since it is a PV bus, the voltage magnitude is adjusted to specified value and V_3^1 is computed as: $V_3^1 = 1.04 \angle 3.077^o$ pu

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}\, V_1^o - Y_{42}\, V_2^1 - Y_{43}\, V_3^1 - Y_{45}\, V_5^0\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{-0.4 + j0.1}{1.0 - j0.0} - \{(-0.39215 + j1.56862) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-0.58823 + j2.35294) \times (0.98140\angle -3.0665^o)\}\right]$$

$$= \frac{0.45293 - j3.8366}{0.98038 - j3.92156} = 0.955715 \angle -7.303^o \text{ pu} = 0.94796 - j0.12149$$

$$V_5^1 = \frac{1}{Y_{55}}\left[\frac{P_5 - jQ_5}{V_5^{o*}} - Y_{51}\, V_1^o - Y_{52}\, V_2^1 - Y_{53}\, V_3^1 - Y_{54}\, V_4^1\right]$$

$$= \frac{1}{Y_{55}}\left[\frac{-0.6 + j0.2}{1.0 - j0.0} - \{(-1.17647 + j4.70588) \times 1.02\angle 0^o\}\right.$$

$$\left. - \{(-1.17647 + j4.70588) \times 1.04\angle 3.077^o\}\right]$$

$$= 0.994618 \angle -1.56^o = 0.994249 - j0.027$$

Thus at end of 1st iteration, we have,

$V_1 = 1.02 \angle 0^0$ pu $\qquad$ $V_2 = 0.98140 \angle -3.066^0$ pu

$V_3 = 1.04 \angle 3.077^0$ pu $\qquad$ $V_4 = 0.955715 \angle -7.303^0$ pu

and $\qquad$ $V_5 = 0.994618 \angle -1.56^0$ pu

Example-3:

Obtain the load flow solution at the end of first iteration of the system with data as given below. The solution is to be obtained for the following cases

(i) All buses except bus 1 are PQ Buses

(ii) Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

(iii) Bus 2 is PV bus, with voltage magnitude specified as 1.04 and $0.25 \le Q_2 \le 1.0$ pu.

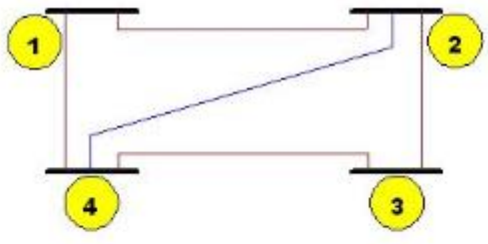


Fig. System for Example 3

Table: Line data of example 3

SB	EB	R (pu)	X (pu)
1	2	0.05	0.15
1	3	0.10	0.30
2	3	0.15	0.45
2	4	0.10	0.30
3	4	0.05	0.15

Table: Bus data of example 3

Bus No.	P_i (pu)	Q_i (pu)	V_i
1	–	–	$1.04\angle 0^0$
2	0.5	−0.2	–
3	−1.0	0.5	–
4	−0.3	−0.1	–

Solution: Note that the data is directly in terms of injected powers at the buses. The bus admittance matrix is formed by inspection as under:

$$Y_{BUS} = \begin{bmatrix} 3.0-j9.0 & -2.0+j6.0 & -1.0+j3.0 & 0 \\ -2.0+j6.0 & 3.666-j11.0 & -0.666+j2.0 & -1.0+j3.0 \\ -1.0+j3.0 & -0.666+j2.0 & 3.666-j11.0 & -2.0+j6.0 \\ 0 & -1.0+j3.0 & -2.0+j6.0 & 3.0-j9.0 \end{bmatrix}$$

Case(i): All buses except bus 1 are PQ Buses

Assume all initial voltages to be $1.0\angle 0^0$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{Y_{22}}\left[\frac{0.5 + j0.2}{1.0 - j0.0} - \{(-2.0 + j6.0)\times(1.04\angle 0^\circ)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^\circ)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^\circ)\}\right]$$

$$= 1.02014 \angle 2.605^\circ$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{o*}} - Y_{31}\, V_1^o - Y_{32}\, V_2^1 - Y_{34}\, V_4^0\right]$$

$$= \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^\circ)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.02014\angle 2.605^\circ)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^\circ)\}\right]$$

$$= 1.03108 \angle -4.831^\circ$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{o*}} - Y_{41}\, V_1^o - Y_{42}\, V_2^1 - Y_{43}\, V_3^1\right]$$

$$= \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.02014\angle 2.605^\circ)\}\right.$$

$$\left. - \{(-2.0 + j6.0)\times(1.03108\angle -4.831^\circ)\}\right]$$

$$= 1.02467 \angle -0.51^\circ$$

Hence

$V_1^1 = 1.04 \angle 0^0$ pu $\qquad V_2^1 = 1.02014 \angle 2.605^0$ pu

$V_3^1 = 1.03108 \angle -4.831^0$ pu $\qquad V_4^1 = 1.02467 \angle -0.51^0$ pu

Case(ii): Bus 2 is a PV bus whose voltage magnitude is specified as 1.04 pu

We first compute Q_2.

$$Q_2 = |V_2|\big[|V_1|(G_{21}\sin\delta_{21} - B_{21}\cos\delta_{21}) + |V_2|(G_{22}\sin\delta_{22} - B_{22}\cos\delta_{22})$$

$$+ |V_3|(G_{23}\sin\delta_{23} - B_{23}\cos\delta_{23}) + |V_4|(G_{24}\sin\delta_{24} - B_{24}\cos\delta_{24})\big]$$

$= 1.04\ [1.04\ \{-6.0\} + 1.04\ \{11.0\} + 1.0\{-2.0\} + 1.0\ \{-3.0\}] = 0.208$ pu.

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{0.5 - j0.208}{1.04\angle 0^\circ} - \{(-2.0 + j6.0)\times(1.04\angle 0^\circ)\}\right.$$

$$\left. - \{(-0.666 + j2.0)\times(1.0\angle 0^\circ)\} - \{(-1.0 + j3.0)\times(1.0\angle 0^\circ)\}\right]$$

$$= 1.051288 + j0.033883$$

The voltage magnitude is adjusted to 1.04. Hence $V_2^1 = 1.04 \angle 1.846^0$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{-1.0 - j0.5}{1.0\angle 0^\circ} - \{(-1.0 + j3.0)\times(1.04\angle 0.0^\circ)\} - \{(-0.666 + j2.0)\times(1.04\angle 1.846^\circ)\} - \{(-2.0 + j6.0)\times(1.0\angle 0^\circ)\}\right]$$

$$= 1.035587\angle -4.951^\circ \text{ pu.}$$

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{0.3 + j0.1}{1.0 - j0.0} - \{(-1.0 + j3.0)\times(1.04\angle 1.846^\circ)\} - \{(-2.0 + j6.0)\times(1.035587\angle -4.951^\circ)\}\right]$$

$$= 0.9985\angle -0.178^0$$

Hence at end of 1st iteration we have:

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.04\ \angle 1.846^0$ pu

$V_3^1 = 1.035587\ \angle -4.951^0$ pu $\qquad V_4^1 = 0.9985\angle -0.178^0$ pu

Case (iii): Bus 2 is PV bus, with voltage magnitude specified as 1.04 & $0.25 \le Q_2 \le 1$ pu. If $0.25 \le Q_2 \le 1.0$ pu then the computed value of $Q_2 = 0.208$ is less than the lower limit. Hence, Q_2 is set equal to 0.25 pu. Iterations are carried out with this value of Q_2. The voltage magnitude at bus 2 can no longer be maintained at 1.04. Hence, there is no necessity to adjust for the voltage magnitude. Proceeding as before we obtain at the end of first iteration,

$V_1^1 = 1.04\ \angle 0^0$ pu $\qquad V_2^1 = 1.05645\ \angle 1.849^0$ pu

$V_3^1 = 1.038546\ \angle -4.933^0$ pu $\qquad V_4^1 = 1.081446\angle 4.896^0$ pu

Limitations of GS load flow analysis:

GS method is very useful for very small systems. It is easily adoptable, it can be generalized and it is very efficient for systems having less number of buses. However, GS LFA fails to converge in systems with one or more of the features as under:

- Systems having large number of radial lines
- Systems with short and long lines terminating on the same bus
- Systems having negative values of transfer admittances
- Systems with heavily loaded lines, etc.

GS method successfully converges in the absence of the above problems. However, convergence also depends on various other set of factors such as: selection of slack bus, initial solution, acceleration factor, tolerance limit, level of accuracy of results needed, type and quality of computer/ software used, etc.

NEWTON –RAPHSON METHOD

Newton-Raphson (NR) method is used to solve a system of non-linear algebraic equations of the form f(x) =0. Consider a set of n non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(25)}$$

Let $x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_n^{\,0}$, be the initial guess of unknown variables and $\Delta x_1^{\,0}, \Delta x_2^{\,0} \ldots\ldots \Delta x_n^{\,0}$ be the respective corrections. Therefore,

$$f_i(x_1^{\,0} + \Delta x_1^{\,0}, x_2^{\,0} + \Delta x_2^{\,0} \ldots\ldots x_n^{\,0} + \Delta x_n^{\,0}) = 0 \qquad i = 1,2\ldots\ldots n \qquad \textbf{(26)}$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^{\,0}, x_2^{\,0} \ldots\ldots x_n^{\,0}) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^{\,0} + \left(\frac{\partial f_i}{\partial x_2}\right) \Delta x_2^{\,0} + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right) \Delta x_n^{\,0}\right]$$

$$+ \text{ Higher order terms} = 0 \qquad \forall\ i = 1,2\ldots\ldots n \qquad \textbf{(27)}$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at ($x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots\ldots x_0^{\,n}$). If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^{\,0} \\ f_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ f_n^{\,0} \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdots & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & & \cdots & \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdots & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^{\,0} \\ \Delta x_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^{\,0} \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $\qquad F^0 = -J^0 \Delta X^0$

Or $\qquad \Delta X^0 = -[J^0]^{-1} F^0 \qquad$ **(29)**

And $\qquad X^1 = X^0 + \Delta X^0 \qquad$ **(30)**

Here, the matrix [J] is called the **Jacobian** matrix. The vector of unknown variables is updated using (30). The process is continued till the difference between two successive iterations is less than the tolerance value.

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus *i*, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus *i*, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \quad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \quad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus *i*

$Q_{i,sp}$ = Specified reactive power at bus *i*

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32) are of the form F(x) = 0. Thus NR method can be applied to solve them. Equations (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix} \quad \textbf{(33)}$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respect to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partial derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \quad \textbf{(34)}$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \quad \textbf{(35)}$$

The powers are computed at any $(r+1)^{st}$ iteration by using the voltages available from previous iteration. The elements of the Jacobian are found using the above equations as:

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|\{G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik}\}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k|(G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J_3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1 \\ k\neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G_{ii}$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2 B_{ii}$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered again as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \frac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

(vi) $M_{ik} = \dfrac{\partial Q_i}{\partial \delta_k} = -(a_k e_i + b_k f_i) = -N_{ik}$

(vii) $L_{ii} = \dfrac{\partial Q_i}{\partial |V_i|}|V_i| = Q_i - B_{ii}|V_i|^2$

(viii) $L_{ik} = \dfrac{\partial Q_i}{\partial |V_k|}|V_k| = a_k f_i - b_k e_i = H_{ik}$

In the above equations,

$Y_{ik} = G_{ik} + jB_{ik}$

$e_k + jf_k = |V_k|(\cos\delta_k + j\sin\delta_k)$

And $a_k + jb_k = (G_{ik} + jB_{ik})(e_k + jf_k)$ **(36)**

If $Y_{ik} = 0.0 + j0.0$ (if there is no line between buses i and k) then the corresponding off-diagonal elements in the Jacobian matrix will also be zero. Hence, the Jacobian is also a sparse matrix.

Size of the sub-matrices of the Jacobian: The dimensions of the various sub-matrices are as per the table below:

Matrix	size
H	$(n_1+n_2)\times(n_1+n_2)$
N	$(n_1+n_2)\times(n_2)$
M	$(n_2)\times(n_1+n_2)$
L	$(n_2)\times(n_2)$
J	$(n_1+2n_2)\times(n_1+2n_2)$
ΔP	$(n_1+n_2)\times 1$
ΔQ	$n_2\times 1$
$\Delta\delta$	$(n_1+n_2)\times 1$
$\Delta\|V\|/\|V\|$	$n_2\times 1$

ALGORITHM FOR NR METHOD
IN POLAR COORDINATES

1. Formulate the YBUS

2. Assume initial voltages as follows:

$$V_i = |V_{i,sp}|\angle 0^\circ \quad \text{(at all PV buses)}$$

$$V_i = 1\angle 0^\circ \quad \text{(at all PQ buses)}$$

3. At $(r+1)^{st}$ iteration, calculate $P_i^{(r+1)}$ at all the PV and PQ buses and $Q_i^{(r+1)}$ at all the PQ buses, using voltages from previous iteration, $V_i^{(r)}$. The formulae to be used are

$$P_{i,Cal} = P_i = G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$Q_{i,Cal} = Q_i = -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

4. Calculate the power mismatches (power residues)

$$\Delta P_i^{(r)} = P_{i,sp} - P_{i,cal}^{(r+1)} \quad \text{(at PV and PQ buses)}$$

$$\Delta Q_i^{(r)} = Q_{i,sp} - Q_{i,cal}^{(r+1)} \quad \text{(at PQ buses)}$$

5. Calculate the Jacobian [$J^{(r)}$] using $V_i^{(r)}$ and its elements spread over H, N, M, L sub- matrices using the relations derived as in (36).

6. Compute

$$\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V|} \end{bmatrix} = [J^{(r)}]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$$

7. Update the variables as follows:

$$\delta_i^{(r+1)} = \delta_i^{(r)} + \Delta\delta_i^{(r)} \quad \text{(at all buses)}$$

$$|V_i|^{(r+1)} = |V_i|^{(r)} + \Delta|V_i|^{(r)}$$

8. Go to step 3 and iterate till the power mismatches are within acceptable tolerance.

DECOUPLED LOAD FLOW

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected power systems, alternative solution methods are possible, taking into account certain observations made of practical systems. These are,

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \frac{\Delta |V|}{|V|} \end{bmatrix} \qquad \textbf{(37)}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta\left|V^{(r)}\right|}{\left|V^{(r)}\right|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1} \Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta\left|V^{(r)}\right|}{\left|V^{(r)}\right|} = \left[L^{(r)}\right]^{-1} \Delta Q^{(r)}$

(v)Update, $\left|V^{(r+1)}\right| = \left|V^{(r)}\right| + \left|\Delta V^{(r)}\right|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

FAST DECOUPLED LOAD FLOW

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$

$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$

The matrix (37) reduces to

$$[\Delta P] = [|V_i||V_j|B'_{ij}][\Delta\delta]$$

$$[\Delta Q] = [|V_i||V_j|B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \tag{38}$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = [B'_{ij}][\Delta\delta]$$

$$\left[\frac{\Delta Q}{|V|}\right] = [B''_{ij}]\left[\frac{\Delta|V|}{|V|}\right] \tag{39}$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

With these assumptions we obtain a loss-less network. In the FDLF method, the matrices $[B']$ and $[B'']$ are constants and need to be inverted only once at the beginning of the iterations.

REPRESENTATION OF TAP CHANGING TRANSFORMERS

Consider a tap changing transformer represented by its admittance connected in series with an ideal autotransformer as shown (a= turns ratio of transformer)

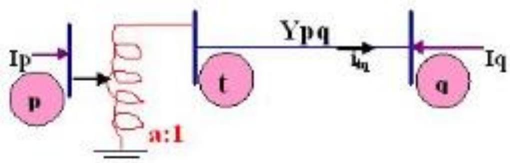


Fig. 2. Equivalent circuit of a tap setting transformer

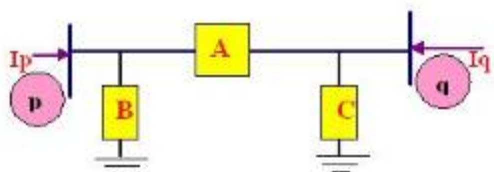


Fig. 3. π-Equivalent circuit of Fig.2 above.

By equating the bus currents in both the mutually equivalent circuits as above, it can be shown that the π-equivalent circuit parameters are given by the expressions as under:

(i) Fixed tap setting transformers (on no load)

A = Ypq/ a

B = 1/a (1/a -1) Ypq

C = (1-1/a) Ypq

(i) Tap changing under load (TCUL) transformers (on load)

A = Ypq

B = (1/a -1) (1/a + 1 – Eq/Ep) Ypq

C = (1-1/a) (Ep/Eq) Ypq

Thus, here, in the case of TCUL transformers, the shunt admittance values are observed to be a function of the bus voltages.

COMPARISON OF LOAD FLOW METHODS

The comparison of the methods should take into account the computing time required for preparation of data in proper format and data processing, programming ease, storage requirements, computation time per iteration, number of iterations, ease and time required for modifying network data when operating conditions change, etc. Since all the methods presented are in the bus frame of reference in admittance form, the data preparation is same for all the methods and the bus admittance matrix can be formed using a simple algorithm, by the rule of inspection. Due to simplicity of the equations, Gauss-Seidel method is relatively easy to program. Programming of NR method is more involved and becomes more complicated if the buses are randomly numbered. It is easier to program, if the PV buses are ordered in sequence and PQ buses are also ordered in sequence.

The storage requirements are more for the NR method, since the Jacobian elements have to be stored. The memory is further increased for NR method using rectangular coordinates. The storage requirement can be drastically reduced by using sparse matrix techniques, since both the admittance matrix and the Jacobian are sparse matrices. The time taken for a single iteration depends on the number of arithmetic and logical operations required to be performed in a full iteration. The Gauss –Seidel method requires the fewest number of operations to complete iteration. In the NR method, the computation of the Jacobian is necessary in every iteration. Further, the inverse of the Jacobian also has to be computed. Hence, the time per iteration is larger than in the GS method and is roughly about 7 times that of the GS method, in large systems, as depicted graphically in figure below. Computation time can be reduced if the Jacobian is updated once in two or three iterations. In FDLF method, the Jacobian is constant and needs to be computed only once. In both NR and FDLF methods, the time per iteration increases directly as the number of buses.

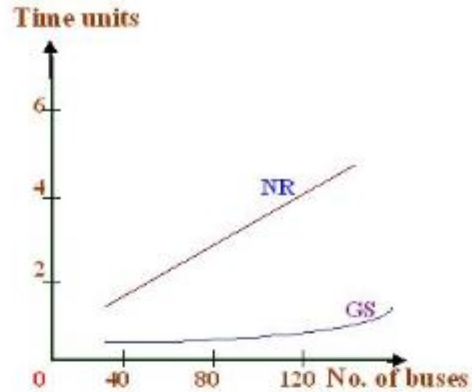


Figure 4. Time per Iteration in GS and NR methods

The number of iterations is determined by the convergence characteristic of the method. The GS method exhibits a linear convergence characteristic as compared to the NR method which has a quadratic convergence. Hence, the GS method requires more number of iterations to get a converged solution as compared to the NR method. In the GS method, the number of iterations increases directly as the size of the system increases. In contrast, the number of iterations is relatively constant in NR and FDLF methods. They require about 5-8 iterations for convergence in large systems. A significant increase in rate of convergence can be obtained in the GS method if an acceleration factor is used. All these variations are shown graphically in figure below. The number of iterations also depends on the required accuracy of the solution. Generally, a voltage tolerance of 0.0001 pu is used to obtain acceptable accuracy and the real power mismatch and reactive power mismatch can be taken as 0.001 pu. Due to these reasons, the NR method is faster and more reliable for large systems. The convergence of FDLF method is geometric and its speed is nearly 4-5 times that of NR method.

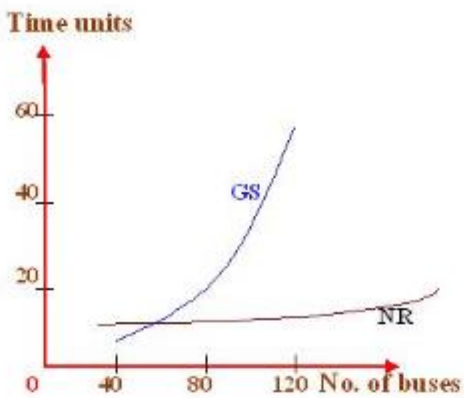


Figure 5. Total time of Iteration in GS and NR methods

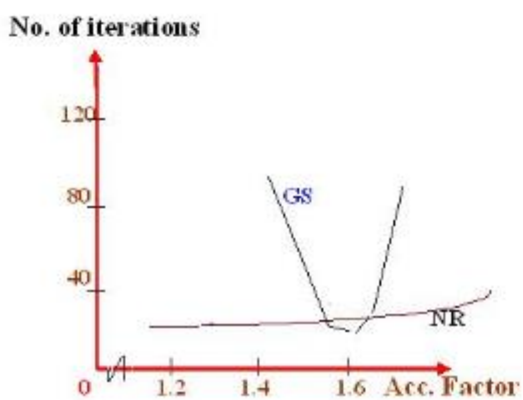


Figure 6. Influence of acceleration factor on load flow methods

FINAL WORD

In this chapter, the load flow problem, also called as the power flow problem, has been considered in detail. The load flow solution gives the complex voltages at all the buses and the complex power flows in the lines. Though, algorithms are available using the impedance form of the equations, the sparsity of the bus admittance matrix and the ease of building the bus admittance matrix, have made algorithms using the admittance form of equations more popular.

The most popular methods are the Gauss-Seidel method, the Newton-Raphson method and the Fast Decoupled Load Flow method. These methods have been discussed in detail with illustrative examples. In smaller systems, the ease of programming and the memory requirements, make GS method attractive. However, the computation time increases with increase in the size of the system. Hence, in large systems NR and FDLF methods are more popular. There is a trade off between various requirements like speed, storage, reliability, computation time, convergence characteristics etc. No single method has all the desirable features. However, NR method is most popular because of its versatility, reliability and accuracy.

CHAPTER 1

REPRESENTATION OF POWER SYSTEMS

[CONTENTS: One line diagram, impedance diagram, reactance diagram, per unit quantities, per unit impedance diagram, formation of bus admittance & impedance matrices, examples]

1.1 One Line Diagram

In practice, electric power systems are very complex and their size is unwieldy. It is very difficult to represent all the components of the system on a single frame. The complexities could be in terms of various types of protective devices, machines (transformers, generators, motors, etc.), their connections (star, delta, etc.), etc. Hence, for the purpose of power system analysis, a simple single phase equivalent circuit is developed called, the one line diagram (OLD) or the single line diagram (SLD). *An SLD is thus, the concise form of representing a given power system.* It is to be noted that a given SLD will contain only such data that are relevant to the system analysis/study under consideration. For example, the details of protective devices need not be shown for load flow analysis nor it is necessary to show the details of shunt values for stability studies.

Symbols used for SLD

Various symbols are used to represent the different parameters and machines as single phase equivalents on the SLD,. Some of the important symbols used are as listed in the table of Figure 1.

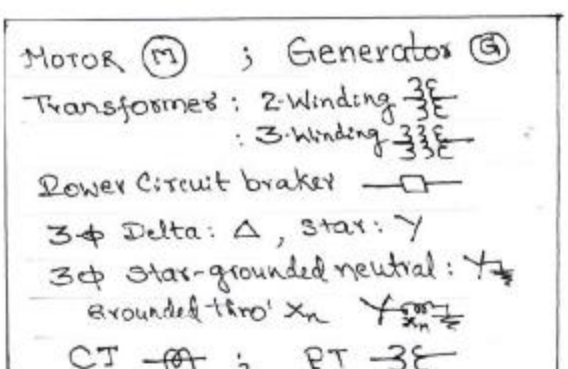
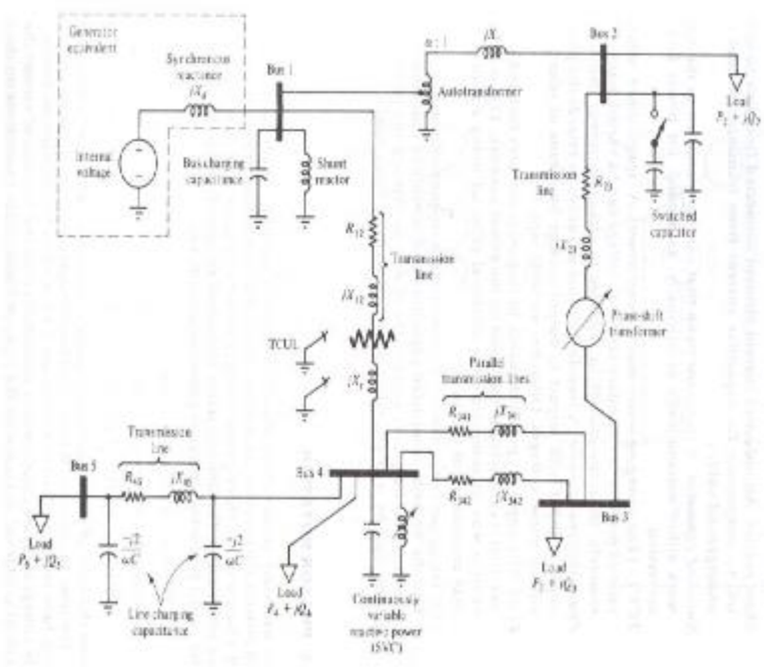


Figure 1. TABLE OF SYMBOLS FOR USE ON SLDS

Example system

Consider for illustration purpose, a sample example power system and data as under: Generator 1: 30 MVA, 10.5 KV, X"= 1.6 ohms, Generator 2: 15 MVA, 6.6 KV, X"= 1.2 ohms, Generator 3: 25 MVA, 6.6 KV, X"= 0.56 ohms, Transformer 1 (3-phase): 15 MVA, 33/11 KV, X=15.2 ohms/phase on HT side, Transformer 2 (3-phase): 15 MVA, 33/6.2 KV, X=16.0 ohms/phase on HT side, Transmission Line: 20.5 ohms per phase, Load A: 15 MW, 11 KV, 0.9 PF (lag); and Load B: 40 MW, 6.6 KV, 0.85 PF (lag). The corresponding SLD incorporating the standard symbols can be shown as in figure 2.



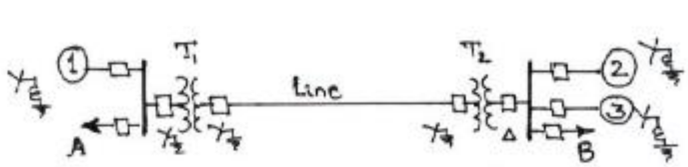


Figure 2. SAMPLE SYSTEM OLD

It is observed here, that the generators are specified in 3-phase MVA, L-L voltage and per phase Y-equivalent impedance, transformers are specified in 3-phase MVA, L-L voltage transformation ratio and per phase Y-equivalent impedance on any one side and the loads are specified in 3-phase MW, L-L voltage and power factor.

1.2 Impedance Diagram

The impedance diagram on single-phase basis for use under balanced conditions can be easily drawn from the SLD. The following assumptions are made in obtaining the impedance diagrams.

Assumptions:

1. The single phase transformer equivalents are shown as ideals with impedances on appropriate side (LV/HV),
2. The magnetizing reactances of transformers are negligible,
3. The generators are represented as constant voltage sources with series resistance or reactance,
4. The transmission lines are approximated by their equivalent π-Models,
5. The loads are assumed to be passive and are represented by a series branch of resistance or reactance and
6. Since the balanced conditions are assumed, the neutral grounding impedances do not appear in the impedance diagram.

Example system

As per the list of assumptions as above and with reference to the system of figure 2, the impedance diagram can be obtained as shown in figure 3.

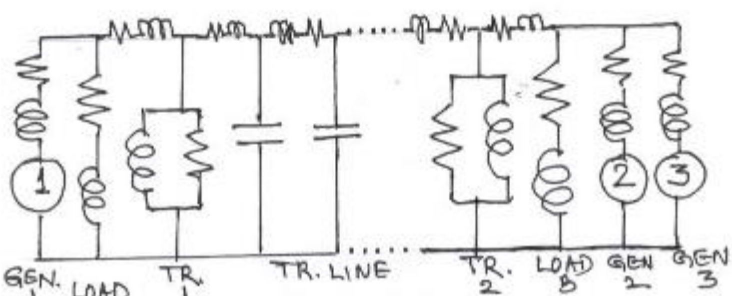


Figure 3. IMPEDANCE DIAGRAM

1.3 Reactance Diagram

With some more additional and simplifying assumptions, the impedance diagram can be simplified further to obtain the corresponding reactance diagram. The following are the assumptions made.

Additional assumptions:

- The resistance is often omitted during the fault analysis. This causes a very negligible error since, resistances are negligible
- Loads are Omitted
- Transmission line capacitances are ineffective &
- Magnetizing currents of transformers are neglected.

Example system

as per the assumptions given above and with reference to the system of figure 2 and figure 3, the reactance diagram can be obtained as shown in figure 4.

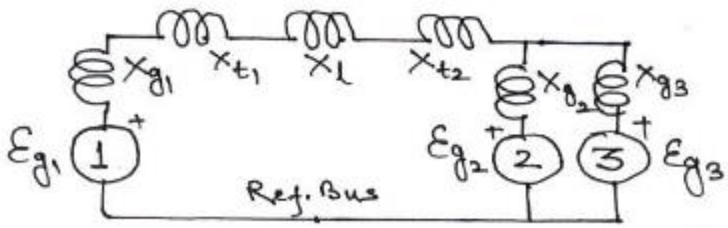


Figure 4. REACTANCE DIAGRAM

Note: These impedance & reactance diagrams are also refered as the *Positive Sequence Diagrams/ Networks.*

1.4 Per Unit Quantities

during the power system analysis, it is a usual practice to represent current, voltage, impedance, power, etc., of an electric power system in per unit or percentage of the base or reference value of the respective quantities. The numerical per unit (pu) value of any quantity is its ratio to a chosen base value of the same dimension. Thus a pu value is a normalized quantity with respect to the chosen base value.

Definition: Per Unit value of a given quantity *is the ratio of the* ***actual value*** *in any given unit to the* ***base value*** *in the same unit.* The percent value is 100 times the pu value. Both the pu and percentage methods are simpler than the use of actual values. Further, the main advantage in using the pu system of computations is that the result that comes out of the sum, product, quotient, etc. of two or more pu values is expressed in per unit

In an electrical power system, the parameters of interest include the current, voltage, complex power (VA), impedance and the phase angle. Of these, the phase angle is dimensionless and the other four quantities can be described by knowing any two of them. Thus clearly, an arbitrary choice of any two base values will evidently fix the other base values.

Normally the nominal voltage of lines and equipment is known along with the complex power rating in MVA. Hence, in practice, the base values are chosen for complex power (MVA) and line voltage (KV). The chosen base MVA is the same for all the parts of the system. However, the base voltage is chosen with reference to a particular section of the system and the other base voltages (with reference to the other sections of the systems, these sections caused by the presence of the transformers) are then related to the chosen one by the turns-ratio of the connecting transformer.

If I_b is the base current in kilo amperes and V_b, the base voltage in kilovolts, then the base MVA is, $S_b = (V_b I_b)$. Then the base values of current & impedance are given by

Base current (kA), $I_b = MVA_b/KV_b$

$$= S_b/V_b \quad (1.1)$$

Base impedance, $Zb = (V_b/I_b)$

$$= (KV_b^2 / MVA_b) \quad (1.2)$$

Hence the per unit impedance is given by

$$Z_{pu} = Z_{ohms}/Z_b$$
$$= Z_{ohms}\,(MVA_b/KV_b^2) \quad (1.3)$$

In 3-phase systems, KV_b is the line-to-line value & MVA_b is the 3-phase MVA. [1-phase MVA = (1/3) 3-phase MVA].

Changing the base of a given pu value:

It is observed from equation (3) that the pu value of impedance is proportional directly to the base MVA and inversely to the square of the base KV. If Z_{pu}^{new} is the pu impedance required to be calculated on a new set of base values: MVA_b^{new} & KV_b^{new} from the already given per unit impedance Z_{pu}^{old}, specified on the old set of base values, MVA_b^{old} & KV_b^{old} , then we have

$$Z_{pu}^{new} = Z_{pu}^{old}\,(MVA_b^{new}/MVA_b^{old})\,(KV_b^{old}/KV_b^{new})^2 \quad (1.4)$$

On the other hand, the change of base can also be done by first converting the given pu impedance to its ohmic value and then calculating its pu value on the new set of base values.

Merits and Demerits of pu System

Following are the advantages and disadvantages of adopting the pu system of computations in electric power systems:

Merits:

- The pu value is the same for both 1-phase and & 3-phase systems
- The pu value once expressed on a proper base, will be the same when refereed to either side of the transformer. Thus the presence of transformer is totally eliminated
- The variation of values is in a smaller range 9nearby unity). Hence the errors involved in pu computations are very less.
- Usually the nameplate ratings will be marked in pu on the base of the name plate ratings, etc.

Demerits:

- If proper bases are not chosen, then the resulting pu values may be highly absurd (such as 5.8 pu, -18.9 pu, etc.). This may cause confusion to the user. However, this problem can be avoided by selecting the base MVA near the high-rated equipment and a convenient base KV in any section of the system.

1.5 pu Impedance / Reactance Diagram

for a given power system with all its data with regard to the generators, transformers, transmission lines, loads, etc., it is possible to obtain the corresponding impedance or reactance diagram as explained above. If the parametric values are shown in pu on the properly selected base values of the system, then the diagram is refered as the per unit impedance or reactance diagram. In forming a pu diagram, the following are the procedural steps involved:

1. Obtain the one line diagram based on the given data
2. Choose a common base MVA for the system
3. Choose a base KV in any one section (Sections formed by transformers)
4. Find the base KV of all the sections present
5. Find pu values of all the parameters: R,X, Z, E, etc.
6. Draw the pu impedance/ reactance diagram.

1.6 Formation Of Y_{BUS} & Z_{BUS}

The performance equations of a given power system can be considered in three different frames of reference as discussed below:

Frames of Reference:

Bus Frame of Reference: There are b independent equations (b = no. of buses) relating the bus vectors of currents and voltages through the bus impedance matrix and bus admittance matrix:

$$E_{BUS} = Z_{BUS}\, I_{BUS}$$

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad (1.5)$$

Branch Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{BR} = Z_{BR}\, I_{BR}$$

$$I_{BR} = Y_{BR}\, E_{BR} \qquad (1.6)$$

Loop Frame of Reference: There are b independent equations (b = no. of branches of a selected Tree sub-graph of the system Graph) relating the branch vectors of currents and voltages through the branch impedance matrix and branch admittance matrix:

$$E_{LOOP} = Z_{LOOP}\, I_{LOOP}$$

$$I_{LOOP} = Y_{LOOP}\, E_{LOOP} \qquad (1.7)$$

Of the various network matrices refered above, the bus admittance matrix (Y_{BUS}) and the bus impedance matrix (Z_{BUS}) are determined for a given power system by the rule of inspection as explained next.

Rule of Inspection

Consider the 3-node admittance network as shown in figure5. Using the basic branch relation: I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: $I_1 = Y_1V_1 + Y_3(V_1 - V_3) + Y_6(V_1 - V_2)$

At node 2: $I_2 = Y_2V_2 + Y_5(V_2 - V_3) + Y_6(V_2 - V_1)$

At node 3: $0 = Y_3(V_3 - V_1) + Y_4V_3 + Y_5(V_3 - V_2)$ (1.8)

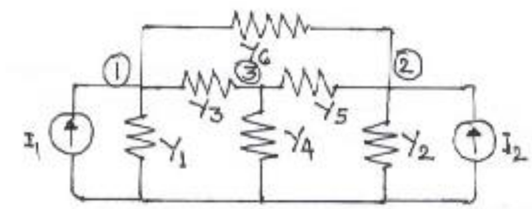


Figure 5. EXAMPLE SYSTEM FOR FINDING Y_{BUS}

These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix} \qquad (1.9)$$

In other words, the relation of equation (9) can be represented in the form

$$I_{BUS} = Y_{BUS}\, E_{BUS} \qquad (1.10)$$

Where, Y_{BUS} is the bus admittance matrix, I_{BUS} & E_{BUS} are the bus current and bus voltage vectors respectively.

By observing the elements of the bus admittance matrix, Y_{BUS} of equation (9), it is observed that the matrix elements can as well be obtained by a simple inspection of the given system diagram:

Diagonal elements: A diagonal element (Y_{ii}) of the bus admittance matrix, Y_{BUS}, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

Off Diagonal elements: An off-diagonal element (Y_{ij}) of the bus admittance matrix, Y_{BUS}, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any.

This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

$$Y_{ii} = \Sigma\, y_{ij} \quad (j = 1,2,\ldots\ldots n)$$
$$Y_{ij} = -\, y_{ij} \quad (j = 1,2,\ldots\ldots n) \qquad (1.11)$$

For i = 1,2,….n, n = no. of buses of the given system, y_{ij} is the admittance of element connected between buses i and j and y_{ii} is the admittance of element connected between bus i and ground (reference bus).

Bus impedance matrix

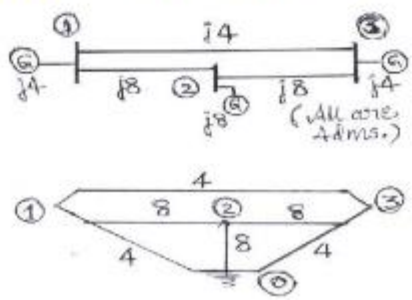
In cases where, the bus impedance matrix is also required, then it cannot be formed by direct inspection of the given system diagram. However, the bus admittance matrix determined by the rule of inspection following the steps explained above, can be inverted to obtain the bus impedance matrix, since the two matrices are inter-invertible. **Note:** It is to be noted that the rule of inspection can be applied only to those power systems that do not have any mutually coupled elements.

1.7 Examples

I EXAMPLES ON RULE OF INSPECTION:

Problem #1: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

$$Y_{BUS} = \begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



II EXAMPLES ON PER UNIT ANALYSIS:

Problem #1:

Two generators rated 10 MVA, 13.2 KV and 15 MVA, 13.2 KV are connected in parallel to a bus bar. They feed supply to 2 motors of inputs 8 MVA and 12 MVA respectively. The operating voltage of motors is 12.5 KV. Assuming the base quantities as 50 MVA, 13.8 KV, draw the per unit reactance diagram. The percentage reactance for generators is 15% and that for motors is 20%.

Solution:

The one line diagram with the data is obtained as shown in figure P1(a).

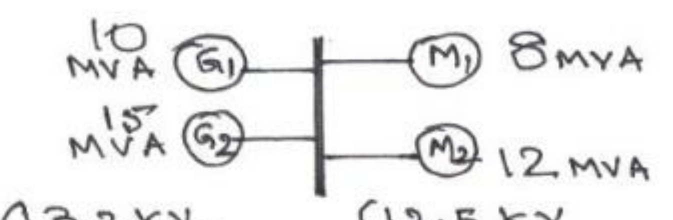


Figure P1(a). OLD of the given system

Selection of base quantities: **50** MVA, **13.8** KV (Given)

Calculation of pu values:

$X_{G1} = j\,0.15\,(50/10)\,(13.2/13.8)^2 = j\,0.6862$ pu.

$X_{G2} = j\,0.15\,(50/15)\,(13.2/13.8)^2 = j\,0.4574$ pu.

$X_{m1} = j\,0.2\,(50/8)\,(12.5/13.8)^2 = j\,1.0256$ pu.

$X_{m2} = j\,0.2\,(50/12)\,(12.5/13.8)^2 = j\,0.6837$ pu.

$E_{g1} = E_{g2} = (13.2/13.8) = 0.9565\,\angle 0^0$ pu

$E_{m1} = E_{m2} = (12.5/13.8) = 0.9058\,\angle 0^0$ pu

Thus the pu reactance diagram can be drawn as shown in figure P1(b).

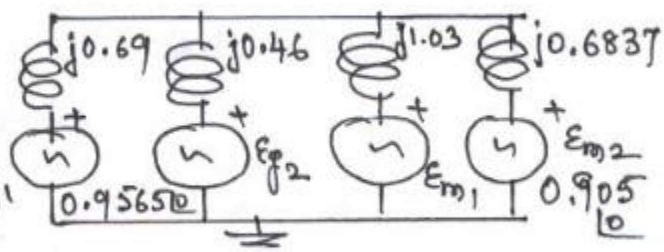


Figure P1(b).
Per Unit Reactance Diagram

Problem #2:

Draw the per unit reactance diagram for the system shown in figure below. Choose a base of 11 KV, 100 MVA in the generator circuit.

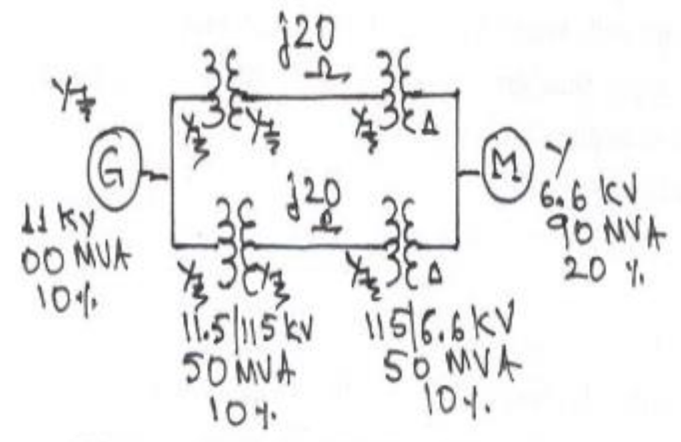


Figure P2(a). OLD of the given system

Solution:

The one line diagram with the data is considered as shown in figure.

Selection of base quantities:

100 MVA, **11** KV in the generator circuit(Given); the voltage bases in other sections are: 11 (115/11.5) = **110** KV in the transmission line circuit and 110 (6.6/11.5) = **6.31** KV in the motor circuit.

Calculation of pu values:

$X_G = j\,0.1$ pu, $X_m = j\,0.2\,(100/90)\,(6.6/6.31)^2 = j\,0.243$ pu.

$X_{t1} = X_{t2} = j\,0.1\,(100/50)\,(11.5/11)^2 = j\,0.2185$ pu.

$X_{t3} = X_{t4} = j\,0.1\,(100/50)\,(6.6/6.31)^2 = j\,0.219$ pu.

$X_{lines} = j\,20\,(100/110^2) = j\,0.1652$ pu.

$E_g = 1.0\angle 0^0$ pu, $E_m = (6.6/6.31) = 1.045\angle 0^0$ pu

Thus the pu reactance diagram can be drawn as shown in figure P2(b).

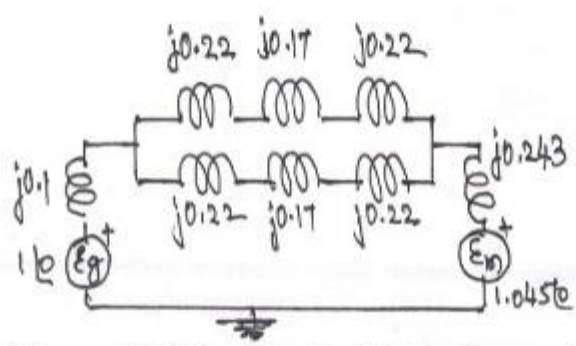


Figure P2(b). Per Unit Reactance Diagram

Problem #3:

A 30 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 15%. The generator supplies 2 motors through a step-up transformer - transmission line – step-down transformer arrangement. The motors have rated inputs of 20 MVA and 10 MVA at 12.8 KV with 20% sub transient reactance each. The 3-phase transformers are rated at 35 MVA, 13.2 KV-Δ /115 KV-Y with 10 % leakage reactance. The line reactance is 80 ohms. Draw the equivalent per unit reactance diagram by selecting the generator ratings as base values in the generator circuit.

Solution:

The one line diagram with the data is obtained as shown in figure P3(a).

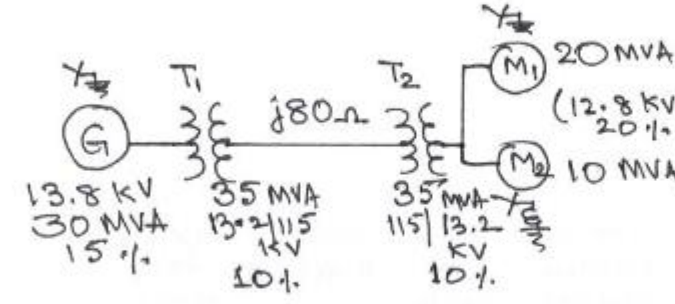


Figure P3(a). OLD of the given system

Selection of base quantities:

30 MVA, **13.8** KV in the generator circuit(Given);

The voltage bases in other sections are:

13.8 (115/13.2) = **120.23** KV in the transmission line circuit and

120.23 (13.26/115) = **13.8** KV in the motor circuit.

Calculation of pu values:

$X_G = j\,0.15$ pu.

$X_{m1} = j\,0.2\,(30/20)\,(12.8/13.8)^2 = j\,0.516$ pu.

$X_{m2} = j\,0.2\,(30/10)\,(12.8/13.8)^2 = j\,0.2581$ pu.

$X_{t1} = X_{t2} = j\,0.1\,(30/35)\,(13.2/13.8)^2 = j\,0.0784$ pu.

$X_{line} = j\,80\,(30/120.23^2) = j\,0.17$ pu.

$E_g = 1.0\angle 0^0$ pu; $E_{m1} = E_{m2} = (6.6/6.31) = 0.93\angle 0^0$ pu

Thus the pu reactance diagram can be drawn as shown in figure P3(b).

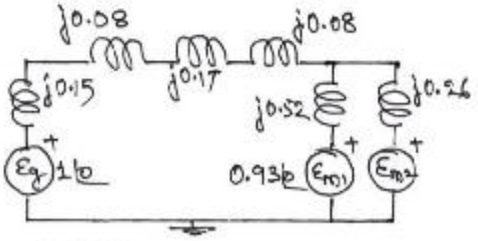


Figure P3(b). Per Unit Reactance Diagram

Problem #4:

A 33 MVA, 13.8 KV, 3-phase generator has a sub transient reactance of 0.5%. The generator supplies a motor through a step-up transformer - transmission line – step-down transformer arrangement. The motor has rated input of 25 MVA at 6.6 KV with 25% sub transient reactance. Draw the equivalent per unit impedance diagram by selecting 25 MVA (3ϕ), 6.6 KV (LL) as base values in the motor circuit, given the transformer and transmission line data as under:

Step up transformer bank: three single phase units, connected Δ–Y, each rated 10 MVA, 13.2/6.6 KV with 7.7 % leakage reactance and 0.5 % leakage resistance;

Transmission line: 75 KM long with a positive sequence reactance of 0.8 ohm/ KM and a resistance of 0.2 ohm/ KM; and

Step down transformer bank: three single phase units, connected Δ–Y, each rated 8.33 MVA, 110/3.98 KV with 8% leakage reactance and 0.8 % leakage resistance;

Solution:

The one line diagram with the data is obtained as shown in figure P4(a).

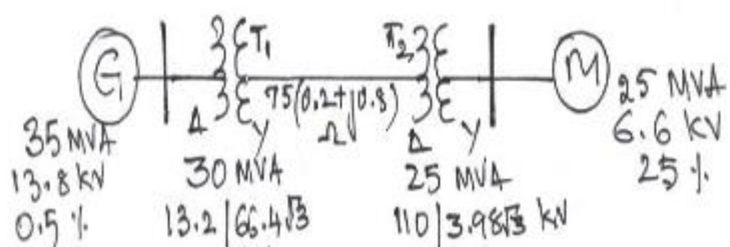


Figure P4(a). OLD of the given system

3-phase ratings of transformers:

T_1: 3(10) = 30 MVA, 13.2/ 66.4√3 KV = 13.2/ 115 KV, X = 0.077, R = 0.005 pu.
T_2: 3(8.33) = 25 MVA, 110/ 3.98√3 KV = 110/ 6.8936 KV, X = 0.08, R = 0.008 pu.

Selection of base quantities:

25 MVA, **6.6** KV in the motor circuit (Given); the voltage bases in other sections are: 6.6 (110/6.8936) = **105.316** KV in the transmission line circuit and 105.316 (13.2/115) = **12.09** KV in the generator circuit.

Calculation of pu values:

$X_m = j\,0.25$ pu; $E_m = 1.0\angle 0^0$ pu.

$X_G = j\,0.005\,(25/33)\,(13.8/12.09)^2 = j\,0.005$ pu; $E_g = 13.8/12.09 = 1.414\angle 0^0$ pu.

$Z_{t1} = 0.005 + j\,0.077\,(25/30)\,(13.2/12.09)^2 = 0.005 + j\,0.0765$ pu. (ref. to LV side)

$Z_{t2} = 0.008 + j\,0.08\,(25/25)\,(110/105.316)^2 = 0.0087 + j\,0.0873$ pu. (ref. to HV side)

$Z_{line} = 75\,(0.2{+}j\,0.8)\,(25/\,105.316^2) = 0.0338 + j\,0.1351$ pu.

Thus the pu reactance diagram can be drawn as shown in figure P4(b).

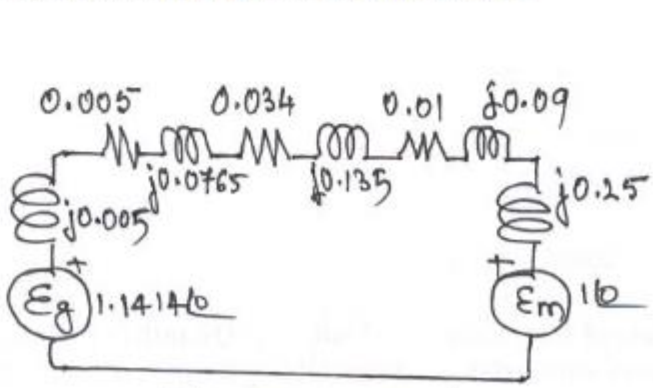


Figure P4(b).
Per Unit Reactance Diagram

CHAPTER 2

SYMMETRICAL THREE PHASE FAULTS

[CONTENTS: Preamble, transients on a transmission line, short circuit of an unloaded synchronous machine- short circuit currents and reactances, short circuit of a loaded machine, selection of circuit breaker ratings, examples]

2.1 Preamble

in practice, any disturbance in the normal working conditions is termed as a FAULT. The effect of fault is to load the device electrically by many times greater than its normal rating and thus damage the equipment involved. Hence all the equipment in the fault line should be protected from being overloaded. In general, overloading involves the increase of current up to 10-15 times the rated value. In a few cases, like the opening or closing of a circuit breaker, the transient voltages also may overload the equipment and damage them.

In order to protect the equipment during faults, fast acting circuit breakers are put in the lines. To design the rating of these circuit breakers or an auxiliary device, the fault current has to be predicted. By considering the equivalent per unit reactance diagrams, the various faults can be analyzed to determine the fault parameters. This helps in the protection and maintenance of the equipment.

Faults can be symmetrical or unsymmetrical faults. In symmetrical faults, the fault quantity rises to several times the rated value equally in all the three phases. For example, a 3-phase fault - a dead short circuit of all the three lines not involving the ground. On the other hand, the unsymmetrical faults may have the connected fault quantities in a random way. However, such unsymmetrical faults can be analyzed by using the *Symmetrical Components*. Further, the neutrals of the machines and equipment may or may not be grounded or the fault may occur through fault impedance. The three-phase fault involving ground is the most severe fault among the various faults encountered in electric power systems.

2.2 Transients on a transmission line

Now, let us Consider a transmission line of resistance R and inductance L supplied by an ac source of voltage v, such that $v = V_m \sin(\omega t+\alpha)$ as shown in figure 1. Consider the short circuit transient on this transmission line. In order to analyze this symmetrical 3-phase fault, the following assumptions are made:

- The supply is a constant voltage source,
- The short circuit occurs when the line is unloaded and

- The line capacitance is negligible.

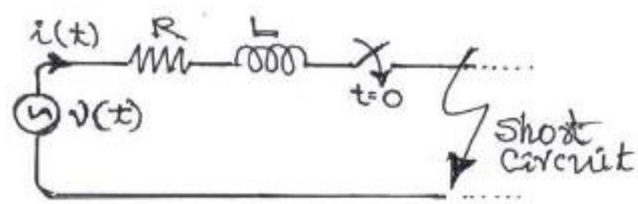


Figure 1. Short Circuit Transients on an Unloaded Line.

Thus the line can be modeled by a lumped R-L series circuit. Let the short circuit take place at t=0. The parameter, α controls the instant of short circuit on the voltage wave. From basic circuit theory, it is observed that the current after short circuit is composed of the two parts as under: $i = i_s + i_t$, Where, i_s is the steady state current and i_t is the transient current. These component currents are determined as follows.

Consider, $v = V_m \sin(\omega t+\alpha)$

$$= iR + L\,(di/dt) \qquad (2.1)$$

and $$i = I_m \sin(\omega t+\alpha-\theta) \qquad (22.)$$

Where $$V_m = \sqrt{2}V;\ I_m = \sqrt{2}I;\ Z_{mag} = \sqrt{[R^2+(\omega L)^2]} = \tan^{-1}(\omega L/R) \qquad (2.3)$$

Thus $$i_s = [V_m/Z] \sin(\omega t+\alpha-\theta) \qquad (2.4)$$

Consider the performance equation of the circuit of figure 1 under circuit as:

$$iR + L\,(di/dt) = 0$$

i.e., $$(R/L + d/dt)i = 0 \qquad (2.5)$$

In order to solve the equation (5), consider the complementary function part of the solution as: $CF = C_1\, e^{(-t/\tau)}$ (2.6)

Where τ (= L/R) is the time constant and C_1 is a constant given by the value of steady state current at t = 0. Thus we have,

$$C_1 = -is(0)$$

$$= -[V_m/Z] \sin(\alpha-\theta)$$

$$= [V_m/Z] \sin(\theta-\alpha) \qquad (2.7)$$

Similarly the expression for the transient part is given by:

$$i_t = -is(0)\, e^{(-t/\tau)}$$

$$= [V_m/Z] \sin(\theta-\alpha)\, e^{(-R/L)t} \qquad (2.8)$$

Thus the total current under short circuit is given by the solution of equation (1) as [combining equations (4) and (8)],

$i = i_s + i_t$

$$= [\sqrt{2}V/Z] \sin(\omega t+\alpha-\theta) + [\sqrt{2}V/Z] \sin(\theta-\alpha)\, e^{(-R/L)t} \qquad (2.9)$$

Thus, i_s is the sinusoidal steady state current called as the *symmetrical short circuit current* and i_t is the unidirectional value called as the *DC off-set current*. This causes the total current to be unsymmetrical till the transient decays, as clearly shown in figure 2.

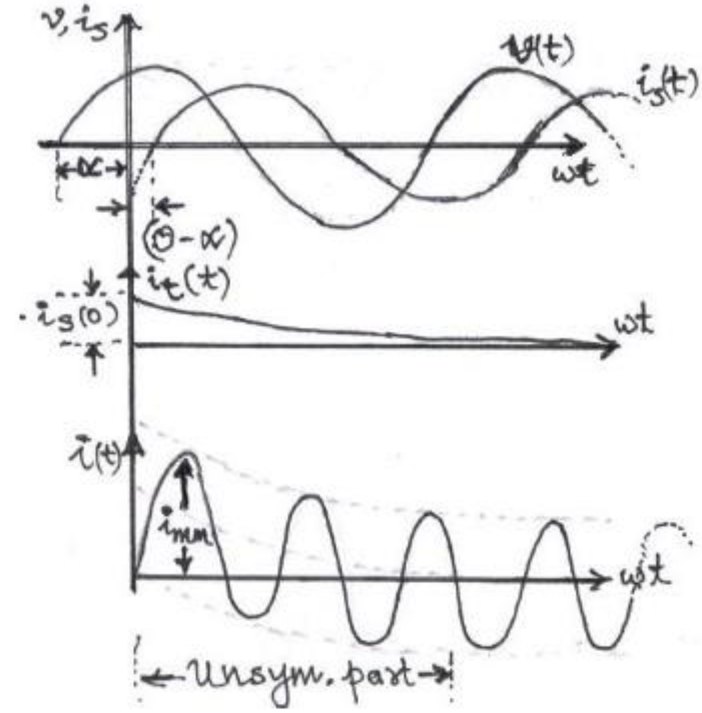


Figure 2. Plot of Symmetrical short circuit current, i(t).

The maximum momentary current, i_{mm} thus corresponds to the first peak. Hence, if the decay in the transient current during this short interval of time is neglected, then we have (sum of the two peak values);

$$i_{mm} = [\sqrt{2}V/Z] \sin(\theta-\alpha) + [\sqrt{2}V/Z] \qquad (2.10)$$

now, since the resistance of the transmission line is very small, the impedance angle θ, can be taken to be approximately equal to 90^0. Hence, we have

$$i_{mm} = [\sqrt{2}V/Z] \cos\alpha + [\sqrt{2}V/Z] \qquad (2.11)$$

This value is maximum when the value of α is equal to zero. This value corresponds to the short circuiting instant of the voltage wave when it is passing through zero. Thus the final expression for the maximum momentary current is obtained as:

$$i_{mm} = 2\,[\sqrt{2}V/Z] \tag{2.12}$$

Thus it is observed that the maximum momentary current is twice the maximum value of symmetrical short circuit current. This is refered as the *doubling effect* of the short circuit current during the symmetrical fault on a transmission line.

2.3 Short circuit of an unloaded synchronous machine

2.3.1 Short Circuit Reactances

Under steady state short circuit conditions, the armature reaction in synchronous generator produces a demagnetizing effect. This effect can be modeled as a reactance, X_a in series with the induced emf and the leakage reactance, X_l of the machine as shown in figure 3. Thus the equivalent reactance is given by:

$$X_d = X_a + X_l \tag{2.13}$$

Where Xd is called as the direct axis *synchronous reactance* of the synchronous machine. Consider now a sudden three-phase short circuit of the synchronous generator on no-load. The machine experiences a transient in all the 3 phases, finally ending up in steady state conditions.

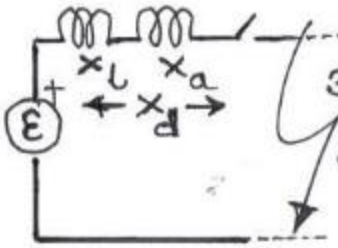


Figure 3. Steady State Short Circuit Model

Immediately after the short circuit, the symmetrical short circuit current is limited only by the leakage reactance of the machine. However, to encounter the demagnetization of the armature short circuit current, current appears in field and damper windings, assisting the rotor field winding to sustain the air-gap flux. Thus during the initial part of the short circuit, there is mutual coupling between stator, rotor and damper windings and hence the corresponding equivalent circuit would be as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d'' = X_l + [1/X_a + 1/X_f + 1/X_{dw}]^{-1} \tag{2.14}$$

Where X_d'' is called as the *sub-transient reactance* of the synchronous machine. Here, the equivalent resistance of the damper winding is more than that of the rotor field winding. Hence, the time constant of the damper field winding is smaller. Thus the damper field effects and the eddy currents disappear after a few cycles.

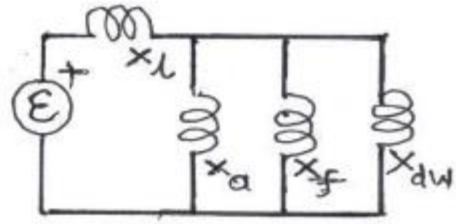


Figure 4. Model during Sub-transient Period of Short Circuit

In other words, X_{dw} gets open circuited from the model of Figure 5 to yield the model as shown in figure 4. Thus the equivalent reactance is given by:

$$X_d' = X_l + [1/X_a + 1/X_f]^{-1} \qquad (2.15)$$

Where X_d' is called as the *transient reactance* of the synchronous machine. Subsequently, X_f also gets open circuited depending on the field winding time constant and yields back the steady state model of figure 3.

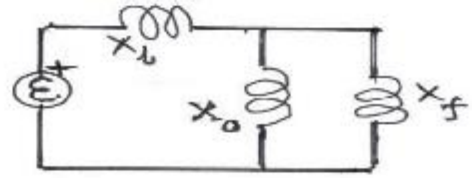


Figure 5. Model during transient Period of Short Circuit

Thus the machine offers a time varying reactance during short circuit and this value of reactance varies from initial stage to final one such that: Xd > Xd' > Xd'

2.3.2 Short Circuit Current Oscillogram

Consider the oscillogram of short circuit current of a synchronous machine upon the occurrence of a fault as shown in figure 6. The symmetrical short circuit current can be divided into three zones: the initial sub transient period, the middle transient period and finally the steady state period. The corresponding reactances, Xd," Xd' and Xd respectively, are offered by the synchronous machine during these time periods.

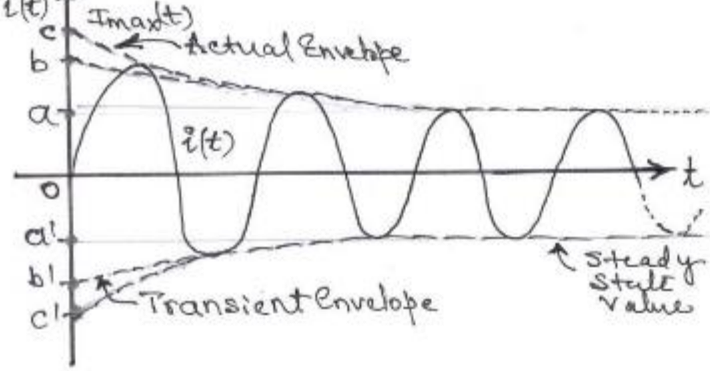


Figure 6. SC current Oscillogram of Armature Current.

The currents and reactances during the three zones of period are related as under in terms of the intercepts on the oscillogram (oa, ob and oc are the y-intercepts as indicated in figure 6):

RMS value of the steady state current = $I = [oa/\sqrt{2}] = [E_g/X_d]$
RMS value of the transient current = $I' = [ob/\sqrt{2}] = [E_g/X_d']$
RMS value of the sub transient current = $I = [oc/\sqrt{2}] = [E_g/X_d'']$ (2.16)

2.4 short circuit of a loaded machine

In the analysis of section 2.3 above, it has been assumed that the machine operates at no load prior to the occurrence of the fault. On similar lines, the analysis of the fault occurring on a loaded machine can also be considered.

Figure 7 gives the circuit model of a synchronous generator operating under steady state conditions supplying a load current I_l to the bus at a terminal voltage V_t. E_g is the induced emf under the loaded conditions and X_d is the direct axis synchronous reactance of the generator.

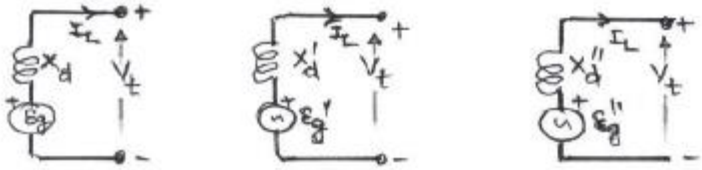


Figure 7. Circuit models for a fault on a loaded machine.

Also shown in figure 7, are the circuit models to be used for short circuit current calculations when a fault occurs at the terminals of the generator, for sub-transient current and transient current values. The induced emf values used in these models are given by the expressions as under:

$$E_g = V_t + j\, I_L X_d = \text{Voltage behind syn. reactance}$$
$$E_g' = V_t + j\, I_L X_d' = \text{Voltage behind transient reactance}$$
$$E_g'' = V_t + j\, I_L X_d'' = \text{Voltage behind subtr. Reactance} \qquad (2.17)$$

The synchronous motors will also have the terminal emf values and reactances. However, then the current direction is reversed. During short circuit studies, they can be replaced by circuit models similar to those shown in figure 7 above, except that the voltages are given by the relations as under:

$$E_m = V_t - j\, I_L X_d = \text{Voltage behind syn. reactance}$$
$$E_m' = V_t - j\, I_L X_d' = \text{Voltage behind transient reactance}$$
$$E_m'' = V_t - j\, I_L X_d'' = \text{Voltage behind subtr. Reactance} \qquad (2.18)$$

The circuit models shown above for the synchronous machines are also very useful while dealing with the short circuit of an interconnected system.

2.5 Selection of circuit breaker ratings

For *selection of circuit breakers*, the maximum momentary current is considered corresponding to its maximum possible value. Later, the current to be interrupted is usually taken as symmetrical short circuit current multiplied by an empirical factor in order to account for the DC off-set current. A value of 1.6 is usually selected as the multiplying factor.

Normally, both the generator and motor reactances are used to determine the momentary current flowing on occurrence of a short circuit. The interrupting capacity of a circuit breaker is decided by X_d'' for the generators and X_d' for the motors.

2.6 Examples

Problem #1: A transmission line of inductance 0.1 H and resistance 5 Ω is suddenly short circuited at $t = 0$, at the far end of a transmission line and is supplied by an ac source of voltage $v = 100 \sin (100\pi t+15^0)$. Write the expression for the short circuit current, i(t). Find the approximate value of the first current maximum for the given values of α and θ. What is this value for α=0, and θ=90^0? What should be the instant of short circuit so that the DC offset current is (i)zero and (ii)maximum?

Solution:

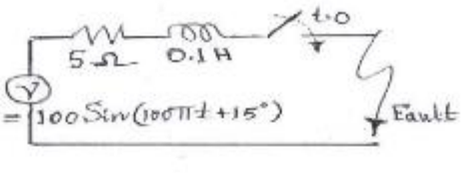


Figure P1.

Consider the expression for voltage applied to the transmission system given by

$$v = V_m \sin(\omega t+\alpha) = 100 \sin(100\pi t+15^0)$$

Thus we get: $V_m = 100$ volts; $f = 50$ Hz and $\alpha = 15^0$.

Consider the impedance of the circuit given by:

$$Z = R + j\omega L = 5 + j(100\pi)(0.1) = 5 + j\,31.416 \text{ ohms.}$$

Thus we have: Z_{mag}=31.8113 Ohms; θ=80.957^0 and τ=L/R=0.1/5=0.02 seconds.

The short circuit current is given by:

$i(t) = [V_m/Z] \sin(\omega t+\alpha-\theta) + [V_m/Z] \sin(\theta-\alpha)\, e^{-(R/L)t}$

$= [100/31.8113]\,[\sin(100\pi t+15^0-80.957^0) + \sin(80.957^0-15^0)\, e^{-(t/0.02)}]$

$= 3.1435 \sin(314.16\,t - 65.96) + 2.871\, e^{-50t}$

Thus we have:

i) $i_{mm} = 3.1435 + 2.871\, e^{-50t}$

where t is the time instant of maximum of symmetrical short circuit current. This instant occurs at $(314.16\, t^c - 65.96^0) = 90^0$; Solving we get, $t = 0.00867$ seconds so that i_{mm} = **5** Amps.

ii) $i_{mm} = 2V_m/Z$ = **6.287** A; for α=0, and $\theta=90^0$ (Also, i_{mm} = 2 (3.1435) = 6.287 A)

iii) DC offset current = $[V_m/Z] \sin(\theta-\alpha)\, e^{-(R/L)t}$

= zero, if $(\theta-\alpha)$ = zero, i.e., $\theta = \alpha$, or $\alpha = \mathbf{80.957^0}$

= maximum if $(\theta-\alpha) = 90^0$, i.e., $\alpha = \theta - 90^0$, or $\alpha = \mathbf{-9.043^0}$.

Problem #2: A 25 MVA, 11 KV, 20% generator is connected through a step-up transformer- T_1 (25 MVA, 11/66 KV, 10%), transmission line (15% reactance on a base of 25 MVA, 66 KV) and step-down transformer-T_2 (25 MVA, 66/6.6 KV, 10%) to a bus that supplies 3 identical motors in parallel (all motors rated: 5 MVA, 6.6 KV, 25%). A circuit breaker-A is used near the primary of the transformer T_1 and breaker-B is used near the motor M_3. Find the symmetrical currents to be interrupted by circuit breakers A and B for a fault at a point P, near the circuit breaker B.

Solution:

Consider the SLD with the data given in the problem statement. The base values are selected as under:

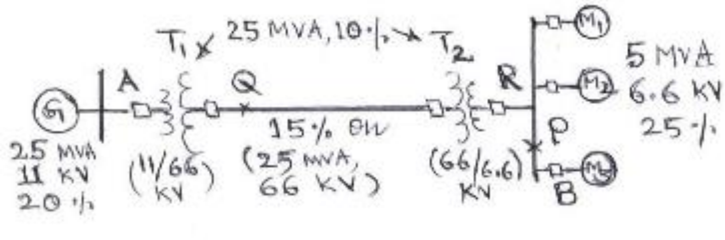


Figure P2(a)

Selection of bases:

S_b = 25 MVA (common); V_b = 11 KV (Gen. circuit)- chosen so that then V_b = 66 KV (line circuit) and V_b = 6.6 KV (Motor circuit).

Pu values:

X_g=j0.2 pu, X_{t1}=X_{t2}=j0.1 pu; X_{m1}=X_{m2}=X_{m3}=j0.25(25/5)=j1.25 pu; X_{line}=j0.15 pu.

Since the system is operating at no load, all the voltages before fault are 1 pu. Considering the pu reactance diagram with the faults at P, we have:

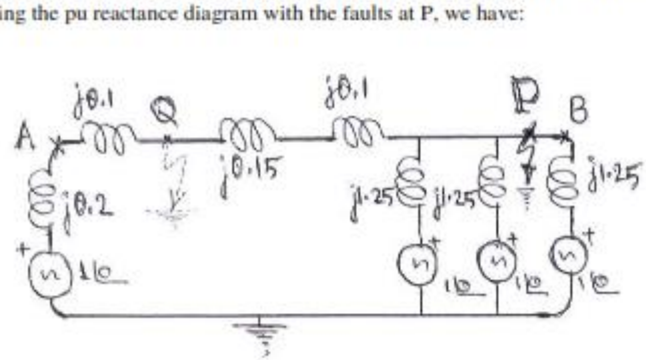


Figure P2(b)

Current to be interrupted by circuit breaker A = 1.0 /j[0.2+0.1+0.15+0.1]

= - j 1.818 pu = - j 1.818 (25/[√3(11)]) = - j 1.818 (1.312) KA = **2.386 KA**

And Current to be interrupted by breaker B = 1/j1.25 = - j 0.8 pu

= - j0.8 (25/[√3(6.6)]) = - j0.8 (2.187) KA = **1.75 KA.**

Problem #3: Two synchronous motors are connected to a large system bus through a short line. The ratings of the various components are: Motors(each)= 1 MVA, 440 volts, 0.1 pu reactance; line of 0.05 ohm reactance and the short circuit MVA at the bus of the large system is 8 at 440 volts. Calculate the symmetrical short circuit current fed into a three-phase fault at the motor bus when the motors are operating at 400 volts.

Solution:

Consider the SLD with the data given in the problem statement. The base values are selected as under:

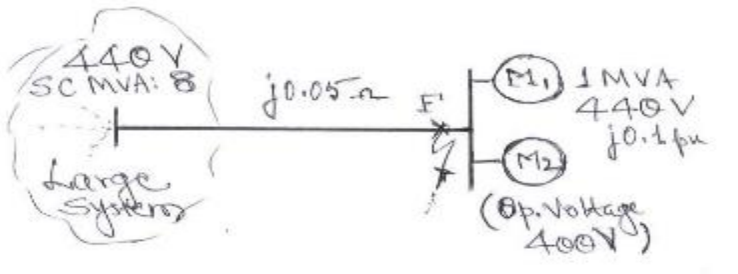


Figure P3.

S_b = 1 MVA; V_b = 0.44 KV (common)- chosen so that X_m(each)=j0.1 pu, Em = $1.0\angle 0^0$, X_{line}=j0.05 $(1/0.44^2)$ = j 0.258 pu and Xlarge-system -= (1/8) = j 0.125 pu.

Thus the prefault voltage at the motor bus; $V_t = 0.4/0.44 = 0.909\angle 0^0$,

Short circuit current fed to the fault at motor bus (I_f = YV);

$$I_f = [0.125 + 0.258]^{-1} + 2.0\ ]0.909 = [20.55 \text{ pu}]\ [1000/(\sqrt{3}(0.4))]$$

$$= 20.55\ (1.312)\ \text{KA} = \mathbf{26.966}\ \text{KA}.$$

Problem #4: A generator-transformer unit is connected to a line through a circuit breaker. The unit ratings are: Gen.: 10 MVA, 6.6 KV, X_d'' = 0.1 pu, X_d' = 0.2 pu and X_d = 0.8 pu; and Transformer: 10 MVA, 6.9/33 KV, X_l = 0.08 pu; The system is operating on no-load at a line voltage of 30 KV, when a three-phase fault occurs on the line just beyond the circuit breaker. Determine the following:

(i) Initial symmetrical RMS current in the breaker,
(ii) Maximum possible DC off-set current in the breaker,
(iii) Momentary current rating of the breaker,
(iv) Current to be interrupted by the breaker and the interrupting KVA and
(v) Sustained short circuit current in the breaker.

Solution:

Consider the base values selected as **10** MVA, **6.6** KV (in the generator circuit) and 6.6(33/6.9) = **31.56** KV(in the transformer circuit). Thus the base current is:

I_b = 10 / [$\sqrt{3}$(31.56)] = **0.183** KA

The pu values are: X_d'' = 0.1 pu, X_d' = 0.2 pu and X_d = 0.8 pu; and $X_{Tr} = 0.08\ (6.9/6.6)^2$ = 0.0874 pu; $V_t = (30/31.6) = 0.95\angle 0^0$ pu.
Initial symmetrical RMS current = $0.95\angle 0^0$ / [0.1 + 0.0874] = 5.069 pu = **0.9277** KA;
Maximum possible DC off-set current = 2 (0.9277) = **1.312** KA;
Momentary current rating = 1.6(0.9277) = **1.4843** KA; (assuming 60% allowance)
Current to be interrupted by the breaker (5 Cycles) = 1.1(0.9277) = **1.0205** KA;
Interrupting MVA = 3(30) (1.0205) = **53.03** MVA;
Sustained short circuit current in the breaker = $0.95\angle 0^0$ (0.183) / [0.8 + 0.0874]
= **0.1959** KA.

CHAPTER 3: SYMMETRICAL COMPONENTS

[CONTENTS: Introduction, The a operator, Power in terms of symmetrical components, Phase shift in Y-Δ transformer banks, Unsymmetrical series impedances, Sequence impedances, Sequence networks, Sequence networks of an unloaded generator, Sequence networks of elements, Sequence networks of power system]

3.1 INTRODUCTION

Power systems are large and complex three-phase systems. In the normal operating conditions, these systems are in balanced condition and hence can be represented as an equivalent single phase system. However, a fault can cause the system to become unbalanced. Specifically, the unsymmetrical faults: open circuit, LG, LL, and LLG faults cause the system to become unsymmetrical. The single-phase equivalent system method of analysis (using SLD and the reactance diagram) cannot be applied to such unsymmetrical systems. Now the question is how to analyze power systems under unsymmetrical conditions? There are two methods available for such an analysis: Kirchhoff's laws method and Symmetrical components method.

The method of symmetrical components developed by C.L. Fortescue in 1918 is a powerful technique for analyzing unbalanced three phase systems. Fortescue defined a linear transformation from phase components to a new set of components called symmetrical components. This transformation represents an unbalanced three-phase system by a set of three balanced three-phase systems. The symmetrical component method is a modeling technique that permits systematic analysis and design of three-phase systems. Decoupling a complex three-phase network into three simpler networks reveals complicated phenomena in more simplistic terms.

Consider a set of three-phase unbalanced voltages designated as V_a, V_b, and V_c. According to Fortescue theorem, these phase voltages can be resolved into following three sets of components.

1. Positive-sequence components, consisting of three phasors equal in magnitude, displaced from each other by 120^0 in phase, and having the same phase sequence as the original phasors, designated as V_{a1}, V_{b1}, and V_{c1}
2. Negative-sequence components, consisting of three phasors equal in magnitude, displaced from each other by 120^0 in phase, and having the phase sequence opposite to that of the original phasors, designated as V_{a2}, V_{b2}, and V_{c2}
3. Zero-sequence components, consisting of three phasors equal in magnitude, and with zero phase displacement from each other, designated as V_{a0}, V_{b0}, and V_{c0}
   Since each of the original unbalanced phasors is the sum of its components, the original phasors expressed in terns of their components are

$$V_a = V_{a1} + V_{a2} + V_{a0}$$
$$V_b = V_{b1} + V_{b2} + V_{b0}$$
$$V_c = V_{c1} + V_{c2} + V_{c0} \qquad (3.1)$$

The synthesis of a set of three unbalanced phasors from the three sets of symmetrical components is shown in Figure1.

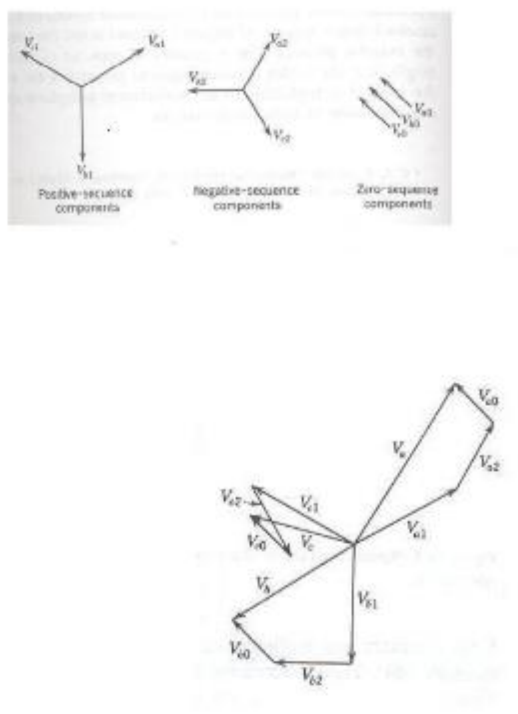


Figure 3.1 Graphical addition of symmetrical components To obtain unbalanced phasors.

3.2 THE OPERATOR 'a'

The relation between the symmetrical components reveals that the phase displacement among them is either 120^0 **or** 0^0**.** Using this relationship, only three independent components is sufficient to determine all the nine components. For this purpose an operator which rotates a given phasor by 120^0 in the positive direction (counterclockwise) is very useful. The letter 'a' is used to designate such a complex operator of unit magnitude with an angle of 120^0. It is defined by

$$a = 1\angle 120^{\circ} = -0.5 + j\,0.866 \qquad (3.2)$$

If the operator 'a' is applied to a phasor twice in succession, the phasor is rotated through 240^0. Similarly, three successive applications of 'a' rotate the phasor through 360^0.

To reduce the number of unknown quantities, let the symmetrical components of V_b and V_c can be expressed as product of some function of the operator a and a component of V_a. Thus,

$$V_{b1} = a^2 V_{a1} \qquad V_{b2} = a V_{a2} \qquad V_{b0} = V_{a0}$$
$$V_{c1} = a\ V_{a1} \qquad V_{c2} = a^2 V_{a2} \qquad V_{c0} = V_{a0}$$

Using these relations the unbalanced phasors can be written as

$$\begin{aligned} V_a &= V_{a0} + V_{a1} + V_{a2} \\ V_b &= V_{a0} + a^2 V_{a1} + a V_{a2} \\ V_c &= V_{a0} + a V_{a1} + a^2 V_{a2} \end{aligned} \tag{3.3}$$

In matrix form,

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix} \tag{3.4}$$

$$\text{Let} \quad Vp = \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix}; \quad Vs = \begin{bmatrix} v_{a0} \\ v_{a1} \\ v_{a2} \end{bmatrix}; \quad A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \tag{3.5}$$

The inverse of A matrix is

$$A^{-1} = \tfrac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \tag{3.6}$$

With these definitions, the above relations can be written as

$$\boldsymbol{V_p = A\ V_s; \qquad V_s = A^{-1} V_p} \tag{3.7}$$

Thus the symmetrical components of V_a, V_b and V_c are given by

$$\begin{aligned} V_{a0} &= 1/3\ (V_a + V_b + V_c) \\ V_{a1} &= 1/3\ (V_a + a\ V_b + a^2 V_c) \\ V_{a2} &= 1/3\ (V_a + a^2 V_b + a V_c) \end{aligned} \tag{3.8}$$

Since the sum of three balanced voltages is zero, the zero-sequence component voltage in a balanced three-phase system is always zero. Further, the sum of line voltages of even an unbalanced three-phase system is zero and hence the corresponding zero-sequence component of line voltages.

NUMERICAL EXAMPLES

Example 1 : The line currents in a 3-ph 4 –wire system are $Ia = 100<30^0$; $Ib = 50<300^0$; $Ic = 30<180^0$. Find the symmetrical components and the neutral current.

Solution:

$$Ia0 = 1/3(Ia + Ib + Ic) \quad = 27.29 < 4.7^0\ A$$
$$Ia1 = 1/3(Ia + a\ Ib + a2Ic) = 57.98 < 43.3^0\ A$$
$$Ia2 = 1/3(Ia + a2\ Ib + a\ Ic) = 18.96 < 24.9^0\ A$$
$$In = Ia + Ib + Ic = 3\ Ia0 = 81.87 < 4.7^0\ A$$

Example 2: The sequence component voltages of phase voltages of a 3-ph system are: Va0 = 100 <00 V; Va1 = 223.6 < -26.60 V ; Va2 = 100 <1800 V. Determine the phase voltages.

Solution:

$$Va = Va0 + Va1 + Va2 \quad = 223.6 < -26.60\ V$$
$$Vb = Va0 + a2Va1 + a\ Va2 = 213 < -99.90\ V$$
$$Vc = Va0 + a\ Va1 + a2\ Va2 = 338.6 < 66.20\ V$$

Example 3: The two seq. components and the corresponding phase voltage of a 3-ph system are $Va0 = 1<-60^0$ V; $Va1 = 2<0^0$ V ; & $Va = 3 <0^0$ V. Determine the other phase voltages.

Solution:

$$Va = Va0 + Va1 + Va2$$
$$Va2 = Va - Va0 - Va1 = 1 < 60^0\ V$$
$$Vb = Va0 + a2Va1 + a\ Va2 = 3 < -120^0\ V$$
$$Vc = Va0 + a\ Va1 + a2\ Va2 = 0\ V$$

Example 4: Determine the sequence components if $Ia = 10<60^0$ A; $Ib = 10<-60^0$ A; $Ic = 10 <180^0$ A.

Solution:

$$Ia0 = 1/3(Ia + Ib + Ic) \quad = 0\ A$$
$$Ia1 = 1/3(Ia + a\ Ib + a2Ic) \quad = 10<60^0\ A$$
$$Ia2 = 1/3(Ia + a2\ Ib + a\ Ic) \quad = 0\ A$$

Observation: If the phasors are balanced, two sequence components will be zero.

Example 5: Determine the sequence components if $Va = 100 <30^0$ V; $Vb = 100 <150^0$ V & $Vc = 100 <-90^0$ V.

Solution:

$$Va0 = 1/3(Va + Vb + Vc) \quad = 0\ V$$
$$Va1 = 1/3(Va + a\ Vb + a2Vc) \quad = 0\ V$$
$$Va2 = 1/3(Va + a2\ Vb + a\ Vc) \quad = 100<30^0\ V$$

Observation: If the phasors are balanced, two sequence components will be zero.

Example 6: The line b of a 3-ph line feeding a balanced Y-load with neutral grounded is open resulting in line currents: $Ia = 10<0^0$ A & $Ic = 10<120^0$ A. Determine the sequence current components.

Solution:

$$\begin{aligned} Ib &= 0\text{ A.} \\ Ia0 &= 1/3(Ia + Ib + Ic) && = 3.33<60^0\text{ A} \\ Ia1 &= 1/3(Ia + a\,Ib + a2Ic) && = 6.66<0^0\text{ A} \\ Ia2 &= 1/3(Ia + a2\,Ib + a\,Ic) && = 3.33<-60^0\text{ A} \end{aligned}$$

Example 7: One conductor of a 3-ph line feeding a balanced delta-load is open. Assuming that line c is open, if current in line a is 10<00 A , determine the sequence components of the line currents.

Solution:

$$\begin{aligned} Ic &= 0\text{ A; } Ia = 10<0^0\text{ A.} && \rightarrow Ib = 10<120^0\text{ A} \\ Ia0 &= 1/3(Ia + Ib + Ic) && = 0\text{ A} \\ Ia1 &= 1/3(Ia + a\,Ib + a2Ic) && = 5.78<-30^0\text{ A} \\ Ia2 &= 1/3(Ia + a2\,Ib + a\,Ic) && = 5.78<30^0\text{ A} \end{aligned}$$

Note: The zero-sequence components of line currents of a delta load (3-ph 3-wire) system are zero.

3.3 POWER IN TERMS OF SYMMETRICAL COMPONENTS

The power in a three-phase system can be expressed in terms of symmetrical components of the associated voltages and currents. The power flowing into a three-phase system through three lines a, b and c is

$$S = P + j\,Q = V_a I_a^* + V_b I_b^* + V_c I_c^* \tag{3.9}$$

where V_a , V_b and V_c are voltages to neutral at the terminals and I_a , I_b, and I_c are the currents flowing into the system in the three lines. In matrix form

$$S = \begin{bmatrix} v_a & v_b & v_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^* = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$

Thus

$$S = [A\,V]^T [AI]^*$$

Using the reversal rule of the matrix algebra

$$S = V^T A^T A^* I^*$$

Noting that $A^T = A$ and a and a^2 are conjugates,

$$S = \begin{bmatrix} v_{a0} & v_{a1} & v_{a2} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

or, since $A^T A^*$ is equal to 3U where U is 3x3 unit matrix

$$S = 3\begin{bmatrix} v_{a0} & v_{a1} & v_{a2} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

Thus the complex three-phase power is given by

$$S = V_a I_a^* + V_b I_b^* + V_c I_c^* = 3\, V_{a0} I_{a0} + 3\, V_{a1} I_{a1} + 3\, V_{a2} I_{a2} \qquad (3.10)$$

Here, $3V_{a0}I_{a0}$, $3V_{a1}I_{a1}$ and $3V_{a2}I_{a2}$ correspond to the three-phase power delivered to the zero-sequence system, positive-sequence system, and negative-sequence system, respectively. Thus, the total three-phase power in the unbalanced system is equal to the sum of the power delivered to the three sequence systems representing the three-phase system.

3.4 PHASE SHIFT OF COMPONENTS IN Y-Δ TRANSFORMER BANKS

The dot convention is used to designate the terminals of transformers. The dots are placed at one end of each of the winding on the same iron core of a transformer to indicate that the currents flowing from the dotted terminal to the unmarked terminal of each winding produces an mmf acting in the same direction in the magnetic circuit. In that case, the voltage drops from dotted terminal to unmarked terminal in each side of the windings are in phase.

The HT terminals of three-phase transformers are marked as H1, H2 and H3 and the corresponding LT side terminals are marked X1, X2 and X3. In Y-Y or Δ-Δ transformers, the markings are such that voltages to neutral from terminals H1, H2, and H3 are in phase with the voltages to neutral from terminals X1, X2, and X3, respectively. But, there will be a phase shift (of 30^0) between the corresponding quantities of the primary and secondary sides of a star-delta (or delta-star) transformer. The standard for connection and designation of transformer banks is as follows:

1. The HT side terminals are marked as H1, H2 and H3 and the corresponding LT side terminals are marked X1, X2 and X3.
2. The phases in the HT side are marked in uppercase letters as A, B, and C. Thus for the sequence abc, A is connected to H1, B to H2 and C to H3. Similarly, the phases in the LT side are marked in lowercase letters as a, b and c.
3. The standard for designating the terminals H1 and X1 on transformer banks requires that the positive-sequence voltage drop from H1 to neutral lead the positive sequence voltage drop from X1 to neutral by 30^0 regardless of the type of connection in the HT

and LT sides. Similarly, the voltage drops from H2 to neutral and H3 to neutral lead their corresponding values, X2 to neutral and X3 to neutral by 30^0.

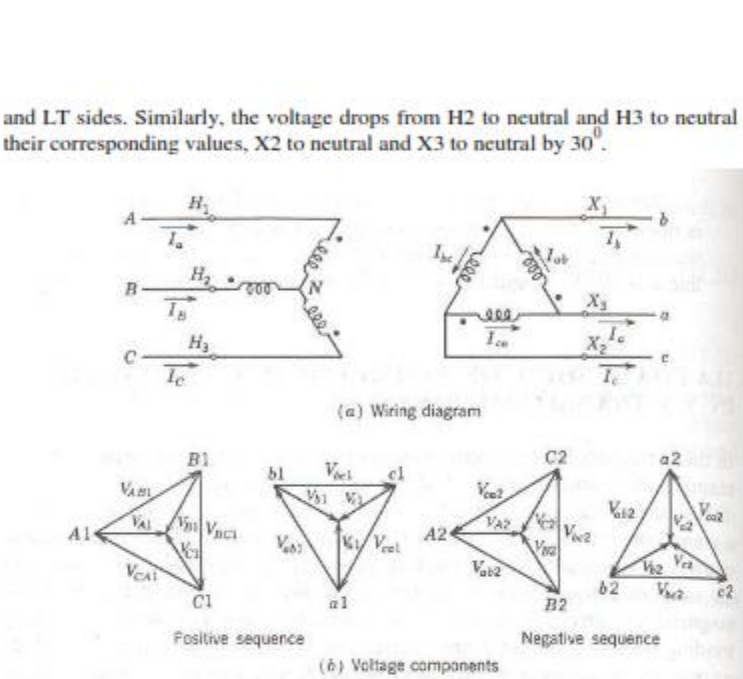


Figure 3.2 Wiring diagram and voltage phasors of a Y-Δ transformer With Y connection on HT side.

Consider a Y- Δ transformer as shown in Figure a. The HT side terminals H1, H2, and H3 are connected to phases A, B, and C, respectively and the phase sequence is ABC. The windings that are drawn in parallel directions are those linked magnetically (by being wound on the same core). In Figure a winding AN is the phase on the Y-side which is linked magnetically with the phase winding bc on the Δ side. For the location of the dots on the windings V_{AN} is in phase with V_{bc}. Following the standards for the phase shift, the phasor diagrams for the sequence components of voltages are shown in Figure b. The sequence component of V_{AN1} is represented as V_{A1} (leaving subscript '$_N$' for convenience and all other voltages to neutral are similarly represented. The phasor diagram reveals that V_{A1} leads V_{b1} by 30^0. This will enable to designate the terminal to which b is connected as X1. Inspection of the positive-sequence and negative-sequence phasor diagrams revels that V_{a1} leads V_{A1} by 90^0 and V_{a2} lags V_{A2} by 90^0.

From the dot convention and the current directions assumed in Figure a, the phasor diagram for the sequence components of currents can be drawn as shown in Figure c. Since the direction specified for I_A in Figure a is away from the dot in the winding and the direction of I_{bc} is also away from the dot in its winding, I_A and I_{bc} are 180^0 out of phase. Hence the phase relation between the Y and Δ currents is as shown in Figure c. From this diagram, it can be seen that I_{a1} leads I_{A1} by 90^0 and I_{a2} lags I_{A2} by 90^0. Summarizing these relations between the symmetrical components on the two sides of the transformer gives:

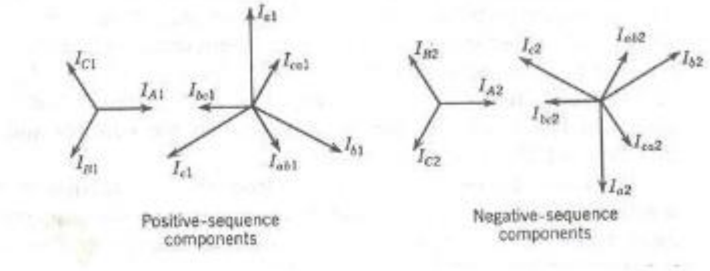


Figure 3.3 Current phasors of Y-Δ transformer with Y connection on HT side.

$$V_{a1} = +j\ V_{A1} \qquad I_{a1} = +j\ I_{A1}$$
$$V_{a2} = -j\ V_{A2} \qquad I_{a1} = -j\ I_{A2} \qquad (3.11)$$

Where each voltage and current is expressed in per unit. Although, these relations are obtained for Y- Δ transformer with Y connection in the HT side, they are valid even when the HT side is connected in Δ and the LT side in Y.

NUMERICAL EXAMPLES

Example 8: Three identical resistors are Y-connected to the LT Y-side of a delta-star transformer. The voltages at the resistor loads are $|V_{ab}| = 0.8$ pu., $|V_{bc}|=1.2$ pu., and $|V_{ca}|=1.0$ pu. Assume that the neutral of the load is not connected to the neutral of the transformer secondary. Find the line voltages on the HT side of the transformer.

Solution:

Assuming an angle of 180^0 for Vca, find the angles of other voltages

$V_{ab} = 0.8\angle 82.8^0$ pu
$V_{bc} = 1.2\angle -41.4^0$ pu
$V_{ca} = 1.0\angle 180^0$ pu

The symmetrical components of line voltages are

$V_{ab0} = 1/3\ (V_{ab} + V_{bc} + V_{ca}) = 0$
$V_{ab1} = 1/3\ (V_{ab} + aV_{bc} + a2V_{ca}) = 0.985\angle 73.6^0$ V
$V_{ab1} = 1/3\ (V_{ab} + a2V_{bc} + aV_{ca}) = 0.235\angle 220.3^0$ V

Since $V_{an1} = V_{ab1}\angle -30^0$ and $V_{an2} = V_{ab2}\angle 30^0$
$V_{an1} = 0.985\angle 73.6^0 - 30^0$
$= 0.985\angle 43.6^0$ pu (L-L base)
$V_{an2} = 0.235\angle 220.3^0 + 30^0$
$= 0.235\angle 250.3^0$ pu(L-L base)

Since each resistor is of $1.0\angle 0$ pu. Impedance,
$I_{an1} = (V_{an1}/Z) = 0.985\angle 43.6^0$ pu.

$I_{an2} = (V_{an2}/Z) = 0.235\angle 250.3^0$ pu.

The directions are +ve for currents from supply toward the delta primary and away from the Y-side toward the load. The HT side line to neutral voltages are

$V_{A1} = -j\, V_{a1} = 0.985\angle -46.4^0$
$V_{A2} = +j\, V_{a2} = 0.235\angle -19.7^0$
$V_A = V_{A1} + V_{A2} = 1.2\angle -41.3^0$ pu.
$V_{B1} = a^2 V_{A1}$ and $V_{B2} = a\, V_{A2}$
$V_B = V_{B1} + V_{B2} = 1\angle 180^0$ pu.
$V_{C1} = a\, V_{A1}$ and $V_{C2} = a^2 V_{A2}$
$V_C = V_{C1} + V_{C2} = 0.8\angle 82.9^0$ pu.

The HT side line voltages are

$V_{AB} = V_A - V_B = 2.06\angle -22.6^0$ pu. (L-N base)
$= (1/3)\, V_{AB} = 1.19\angle -22.6^0$ pu. (L-L base)
$V_{BC} = V_B - V_c = 1.355\angle 215.8^0$ pu. (L-N base)
$= (1/3)\, V_{BC} = 0.782\angle 215.8^0$ pu. (L-L base)
$V_{CA} = V_C - V_A = 1.78\angle 116.9^0$ pu. (L-N base)
$= (1/3)\, V_{CA} = 1.028\angle 116.9^0$ pu. (L-L base)

3.5 UNSYMMETRICAL IMPEDANCES

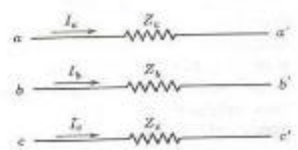


Figure 3.4 Portion of three-phase system representing three unequal series impedances.

Consider the network shown in Figure. Assuming that there is no mutual impedance between the impedances Za, Zb, and Zc, the voltage drops Vaa', vbb', and Vcc' can be expressed in matrix form as

$$\begin{bmatrix} V_{aa'} \\ V_{bb'} \\ V_{cc'} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \tag{3.12}$$

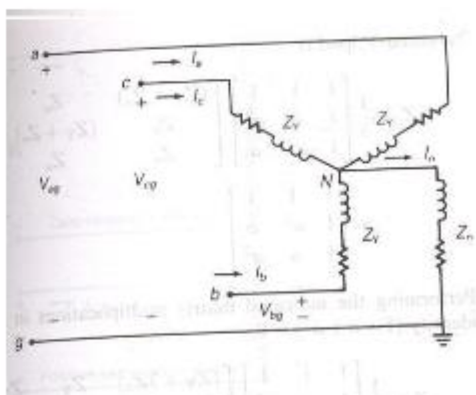
And in terms of symmetrical components of voltage and current as

$$A \begin{bmatrix} V_{aa'0} \\ V_{aa'1} \\ V_{aa'2} \end{bmatrix} = \begin{bmatrix} Z_a & 0 & 0 \\ 0 & Z_b & 0 \\ 0 & 0 & Z_c \end{bmatrix} A \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \tag{3.13}$$

If the three impedances are equal (i.e., if Za = Zb = Zc), Eq reduces to

$$V_{aa'1} = Z_a I_{a1}; \qquad V_{aa'2} = Z_a I_{a2}; \qquad V_{aa'0} = Z_a I_{a0} \tag{3.14}$$

Thus, the symmetrical components of unbalanced currents flowing in balanced series impedances (or in a balanced Y load) produce voltage drops of like sequence only. However, if the impedances are unequal or if there exists mutual coupling, then voltage drop of any one sequence is dependent on the currents of all the sequences.



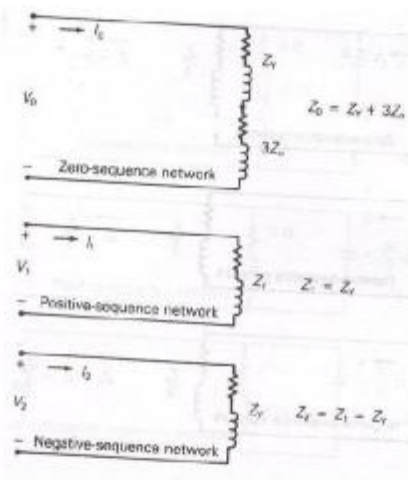


Figure 3.5 Sequence impedances of a Y-connected load.

NUMERICAL EXAMPLES

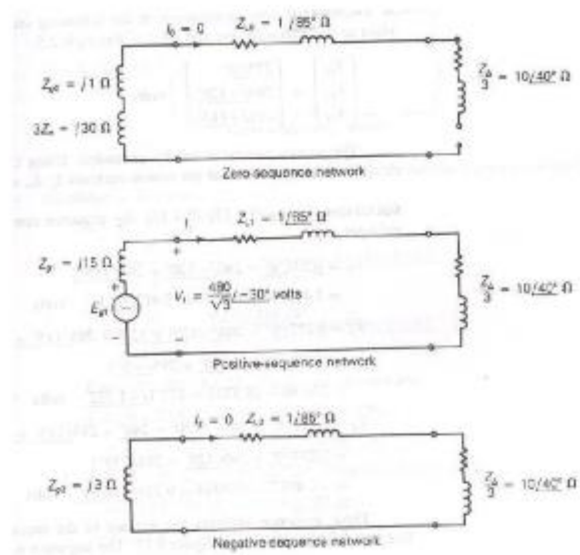
Example 9: A Y-connected source with phase voltages Vag = 277<0^0, Vbg = 260<-120^0 and Vcg = 295<115^0 is applied to a balanced Δ load of 30<40^0 Ω/phase through a line of impedance 1<85^0 Ω. The neutral of the source is solidly grounded. Draw the sequence networks of the system and find source currents.

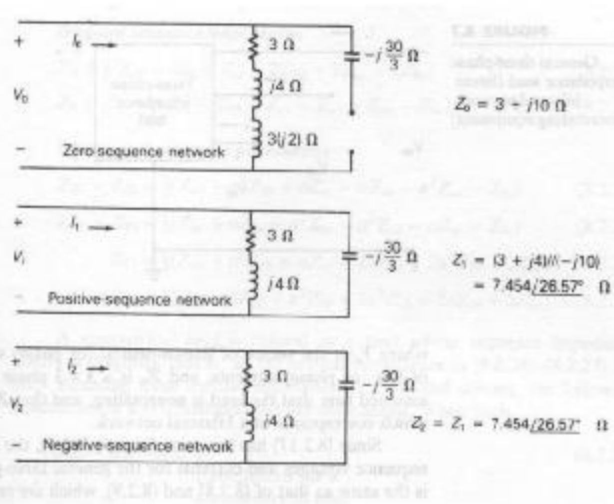
Solution:

Va0 = 15.91<62.110 V
Va1 = 277.1<-1.70 V
Va2 = 9.22<216.70 V
Y eq. of Δ load = 10<400 Ω/phase
Zline = 1<850 Ω.
Zneutral = 0

Ia0 = 0<00 A
Ia1 = 25.82<-45.60 A
Ia2 = 0.86<172.80 A

Ia = 25.15<-46.80 A
Ib = 25.71<196.40 A
Ic = 26.62<73.80 A





3.6 SEQUENCE IMPEDANCES AND SEQUENCE NETWORKS

The impedance of a circuit to positive-sequence currents alone is called the impedance to positive-sequence current or simply *positive-sequence impedance*, which is generally denoted as Z_1. Similarly, the impedance of a circuit to negative-sequence currents alone is called the impedance to negative-sequence current or simply *negative-sequence* *impedance*, which is generally denoted as Z_2. The impedance of a circuit to zero-sequence currents alone is called the impedance to zero-sequence current or simply *zero-sequence impedance*, which is generally denoted as Z_0. In the analysis of an unsymmetrical fault on a symmetrical system, the symmetrical components of the unbalanced currents that are flowing are determined. Since in a balanced system, the components currents of one sequence cause voltage drops of like sequence only and are independent of currents of other sequences, currents of any one sequence may be considered to flow in an independent network composed of the generated voltages, if any, and impedances to the current of that sequence only.

The single-phase equivalent circuit consisting of the impedances to currents of any one sequence only is called the *sequence network* of that particular sequence. Thus, the sequence network corresponding to positive-sequence current is called the *positive-sequence network*. Similarly, the sequence network corresponding to negative-sequence current is called *negative-sequence network*, and that corresponding to zero-sequence current is called *zero-sequence network*. The sequence networks are interconnected in a particular way to represent various unsymmetrical fault conditions. Therefore, to calculate the effect of a fault by the method of symmetrical components, it is required to determine the sequence networks.

3.7 SEQUENCE NETWORKS OF UNLOADED GENERATOR

Consider an unloaded generator which is grounded through a reactor as shown in Figure. When a fault occurs, unbalanced currents depending on the type of fault will flow through the lines. These currents can be resolved into their symmetrical components. To draw the sequence networks of this generator, the component voltages/currents, component impedances are to be determined. The generated voltages are of positive-sequence only as the generators are designed to supply balanced three-phase voltages. Hence, positive-sequence network is composed of an emf in series with the positive-sequence impedance. The generated emf in this network is the no-load terminal voltage to neutral, which is also equal to the transient and subtransient voltages as the generator is not loaded. The reactance in this network is the subtransient, transient, or synchronous reactance, depending on the condition of study.

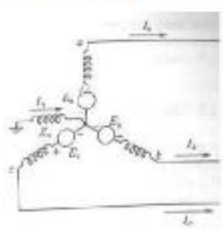


Figure 3.6 Circuit of an unloaded generator grounded through reactance.

The negative- and zero-sequence networks are composed of only the respective sequence impedances as there is no corresponding sequence emf. The reference bus for the positive- and negative-sequence networks is the neutral of the generator.

The current flowing in the impedance Zn between neutral and ground is $3I_{a0}$ as shown in Fig. 3.6. Thus the zero-sequence voltage drop from point a to the ground, is given by: ($-I_{a0}Z_{g0} - 3I_{a0}Zn$), where Z_{g0} is the zero-sequence impedance of the generator. Thus the zero-sequence network, which is single-phase equivalent circuit assumed to carry only one phase, must have an zero-sequence impedance of $Zo = (Z_{g0} + 3Zn)$.

From the sequence networks, the voltage drops from point a to reference bus (or ground) are given by

$$\begin{aligned} V_{a1} &= Ea - I_{a1}Z_1 \\ V_{a2} &= \quad\;\, - I_{a2}Z_2 \\ V_{a0} &= \quad\;\, - I_{a0}\,Z_0 \end{aligned} \tag{3.15}$$

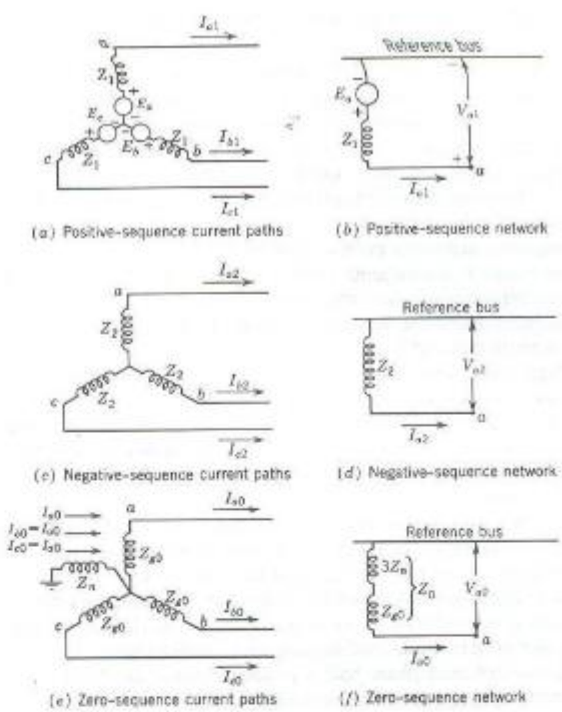


Figure 3.7 Sequence current paths in a generator and The corresponding sequence networks.

Eq. 3.15 applicable to any unloaded generator are valid for loaded generator under steady state conditions. These relations are also applicable for transient or subtransient conditions of a loaded generator if Eg' or Eg" is substituted for Ea.

3.8 SEQUENCE IMPEDANCE OF CIRCUIT ELEMENTS

For obtaining the sequence networks, the component voltages/ currents and the component impedances of all the elements of the network are to be determined. The usual elements of a power system are: passive loads, rotating machines (generators/ motors), transmission lines and transformers. The positive- and negative-sequence impedances of linear, symmetrical, static circuits are identical (because the impedance of such circuits is independent of phase order provided the applied voltages are balanced).

The sequence impedances of rotating machines will generally differ from one another. This is due to the different conditions that exists when the sequence currents flows. The flux due to negative-sequence currents rotates at double the speed of rotor while that the positive-sequence currents is stationary with respect to the rotor. The resultant flux due to zero-sequence currents is ideally zero as these flux components adds up to zero, and hence the zero-sequence reactance is only due to the leakage flux. Thus, the zero-sequence impedance of these machines is smaller than positive- and negative-sequence impedances.

The positive- and negative-sequence impedances of a transmission line are identical, while the zero-sequence impedance differs from these. The positive- and negative-sequence impedances are identical as the transposed transmission lines are balanced linear circuits. The zero-sequence impedance is higher due to magnetic field set up by the zero-sequence currents is very different from that of the positive- or negative-sequence currents (because of no phase difference). The zero-sequence reactance is generally 2 to 3.5 times greater than the positive- sequence reactance. It is customary to take all the sequence impedances of a transformer to be identical, although the zero-sequence impedance slightly differs with respect to the other two.

3.9 SEQUENCE NETWORKS OF POWER SYSTEMS

In the method of symmetrical components, to calculate the effect of a fault on a power system, the sequence networks are developed corresponding to the fault condition. These networks are then interconnected depending on the type of fault. The resulting network is then analyzed to find the fault current and other parameters.

Positive- and Negative-Sequence Networks: The positive-sequence network is obtained by determining all the positive-sequence voltages and positive-sequence impedances of individual elements, and connecting them according to the SLD. All the generated emfs are positive-sequence voltages. Hence all the per unit reactance/impedance diagrams obtained in the earlier chapters are positive-sequence networks. The negative-sequence generated emfs are not present. Hence, the negative-sequence network for a power system is obtained by omitting all the generated emfs (short circuiting emf sources) and replacing all impedances by negative-sequence impedances from the positive-sequence networks.

Since all the neutral points of a symmetrical three-phase system are at the same potential when balanced currents are flowing, the neutral of a symmetrical three-phase system is the logical reference point. It is therefore taken as the reference bus for the positive- and negative-sequence networks. Impedances connected between the neutral of the machine and ground is not a part of either the positive- or negative- sequence networks because neither positive- nor negative-sequence currents can flow in such impedances.

Zero-Sequence Networks: The zero-sequence components are the same both in magnitude and in phase. Thus, it is equivalent to a single-phase system and hence, zero-sequence currents will flow only if a return path exists. The reference point for this network is the ground (Since zero-sequence currents are flowing, the ground is not necessarily at the same point at all points and the reference bus of zero-sequence network does not represent a ground of uniform potential. The return path is conductor of zero impedance, which is the reference bus of the zero-sequence network.).

If a circuit is Y-connected, with no connection from the neutral to ground or to another neutral point in the circuit, no zero-sequence currents can flow, and hence the impedance to zero-sequence current is infinite. This is represented by an open circuit between the neutral of the Y-connected circuit and the reference bus, as shown in Fig. 3.8a. If the neutral of the Y-connected circuit is grounded through zero impedance, a zero-impedance path (short circuit) is connected between the neutral point and the reference bus, as shown in Fig. 3.8b. If an impedance Zn is connected between the neutral and the ground of a Y-connected circuit, an impedance of 3Zn must be connected between the neutral and the reference bus (because, all the three zero-sequence currents ($3I_{a0}$) flows through this impedance to cause a voltage drop of $3I_{a0}\,Z_0$), as shown in Fig. 3.8c.

A Δ-connected circuit can provide no return path; its impedance to zero-sequence line currents is therefore infinite. Thus, the zero-sequence network is open at the Δ-connected circuit, as shown in Fig.3.9 However zero-sequence currents can circulate inside the Δ-connected circuit.

The zero-sequence equivalent circuits of three-phase transformers deserve special attention. The different possible combinations of the primary and the secondary windings in Y and Δ alter the zero-sequence network. The five possible connections of two-winding transformers and their equivalent zero-sequence networks are shown in Fig.3.10. The networks are drawn remembering that there will be no primary current when there is no secondary current, neglecting the no-load component. The arrows on the connection diagram show the possible paths for the zero-sequence current. Absence of an arrow indicates that the connection is such that zero-sequence currents cannot flow. The letters P and Q identify the corresponding points on the connection diagram and equivalent circuit:

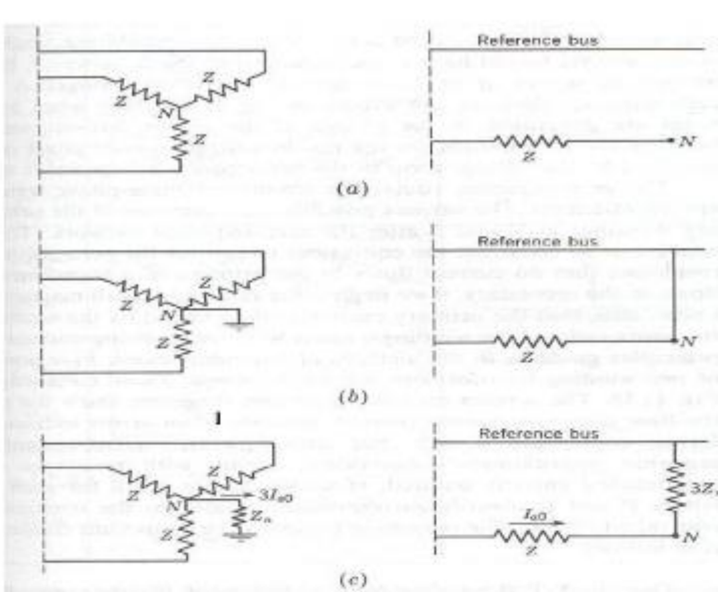


Figure 3.8 Zero-sequence equivalent networks of Y-connected load

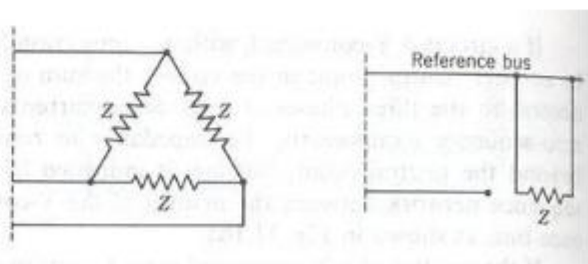


Figure 3.9 Zero-sequence equivalent networks of Δ -connected load

1. *Case 1: Y-Y Bank with one neutral grounded:* If either one of the neutrals of a Y-Y bank is ungrounded, zero-sequence current cannot flow in either winding (as the absence of a path through one winding prevents current in the other). An open circuit exists for zero-sequence current between two parts of the system connected by the transformer bank.
2. *Case 2: Y-Y Bank with both neutral grounded*: In this case, a path through transformer exists for the zero-sequence current. Hence zero-sequence current can flow in both sides of the transformer provided there is complete outside closed path for it to flow. Hence the points on the two sides of the transformer are connected by the zer0-sequence impedance of the transformer.

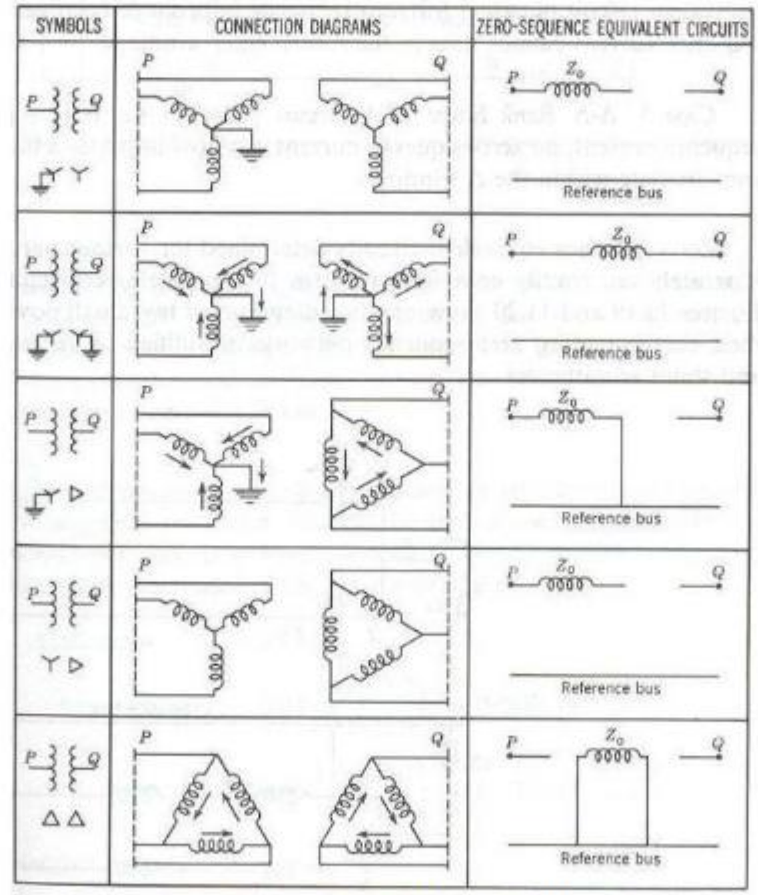


Figure 3.10 Zero-sequence equivalent networks of three-phase transformer banks for various combinations.

3. *Case 3: Y- Δ Bank with grounded Y:* In this case, there is path for zero-sequence current to ground through the Y as the corresponding induced current can circulate in the Δ. The equivalent circuit must provide for a path from lines on the Y side through zero-sequence impedance of the transformer to the reference bus. However, an open circuit must exist between line and the reference bus on the Δ side. If there is an impedance Zn between neutral and ground, then the zero-sequence impedance must include 3Zn along with zero-sequence impedance of the transformer.

4. *Case 4: Y- Δ Bank with ungrounded Y:* In this case, there is no path for zero-sequence current. The zero-sequence impedance is infinite and is shown by an open circuit.
5. *Case 5: Δ-Δ Bank:* In this case, there is no return path for zero-sequence current. The zero-sequence current cannot flow in lines although it can circulate in the Δ windings.
6. The zero-sequence equivalent circuits determined for the individual parts separately are connected according to the SLD to form the complete zero-sequence network.

Procedure to draw the sequence networks

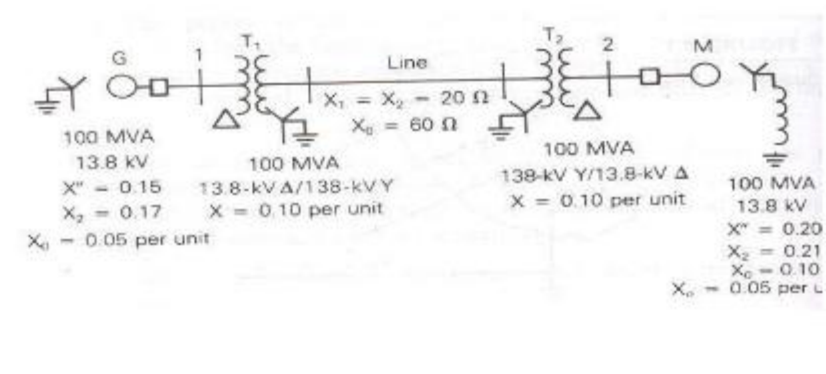
The sequence networks are three separate networks which are the single-phase equivalent of the corresponding symmetrical sequence systems. These networks can be drawn as follows:

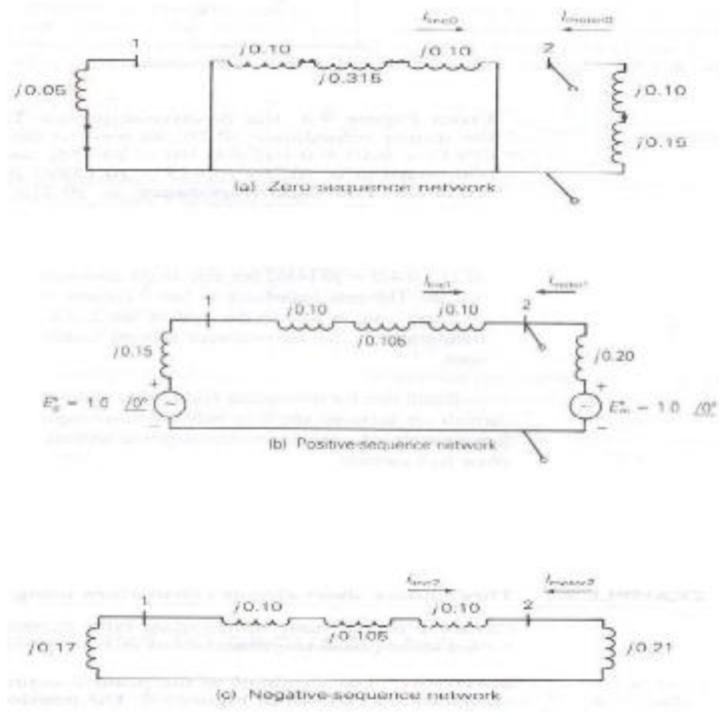
1. For the given condition (steady state, transient, or subtransient), draw the reactance diagram (selecting proper base values and converting all the per unit values to the selected base, if necessary). This will correspond to the positive-sequence network.

2. Determine the per unit negative-sequence impedances of all elements (if the values of negative sequence is not given to any element, it can approximately be taken as equal to the positive-sequence impedance). Draw the negative-sequence network by replacing all emf sources by short circuit and all impedances by corresponding negative-sequence impedances in the positive-sequence network.
3. Determine the per unit zero-sequence impedances of all the elements and draw the zero-sequence network corresponding to the grounding conditions of different elements.

NUMERICAL EXAMPLES

Example 10: For the power system shown in the SLD, draw the sequence networks.



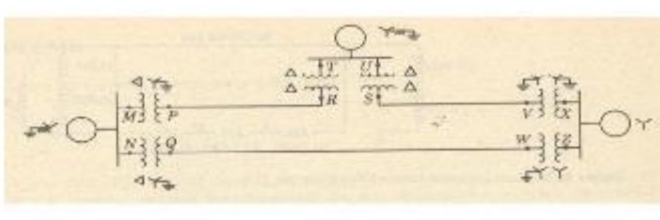


(a) Zero sequence network

(b) Positive-sequence network

(c) Negative-sequence network

EXERCISE PROBLEM: For the power system shown in the SLD, draw the sequence networks.



CHAPTER 4: UNSYMMETRICAL FAULTS

[CONTENTS: Preamble, L-G, L-L, L-L-G and 3-phase faults on an unloaded alternator without and with fault impedance, faults on a power system without and with fault impedance, open conductor faults in power systems, examples]

4.1 PREAMBLE

The unsymmetrical faults will have faulty parameters at random. They can be analyzed by using the symmetrical components. The standard types of unsymmetrical faults considered for analysis include the following (in the order of their severity):

- Line–to–Ground (L-G) Fault
- Line–to–Line (L-L) Fault
- Double Line–to–Ground (L-L-G)Fault and
- Three-Phase–to–Ground (LLL-G) Fault.

Further the neutrals of various equipment may be grounded or isolated, the faults can occur at any general point F of the given system, the faults can be through a fault impedance, etc. Of the various types of faults as above, the 3-ϕ fault involving the ground is the most severe one. Here the analysis is considered in two stages as under: (i) Fault at the terminals of a Conventional (Unloaded) Generator and (ii) Faults at any point F, of a given Electric Power System (EPS).

Consider now the symmetrical component relational equations derived from the three sequence networks corresponding to a given unsymmetrical system as a function of sequence impedances and the positive sequence voltage source in the form as under:

$$\begin{aligned} V_{a0} &= -I_{a0}Z_0 \\ V_{a1} &= E_a - I_{a1}Z_1 \\ V_{a2} &= -I_{a2}Z_2 \end{aligned} \qquad (4.1)$$

These equations are refered as the *sequence equations*. In matrix Form the sequence equations can be considered as:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \qquad (4.2)$$

This equation is used along with the equations i.e., conditions under fault (c.u.f.), derived to describe the fault under consideration, to determine the sequence current I_{a1} and hence the fault current I_f, in terms of E_a and the sequence impedances, Z_1, Z_2 and Z_0. Thus during unsymmetrical fault analysis of any given type of fault, two sets of equations as follows are considered for solving them simultaneously to get the required fault parameters:

- Equations for the conditions under fault (c.u.f.)

➢ Equations for the sequence components (sequence equations) as per (4.2) above.

4.2 SINGLE LINE TO GROUND FAULT ON A CONVENTIONAL (UNLOADED) GENERATOR

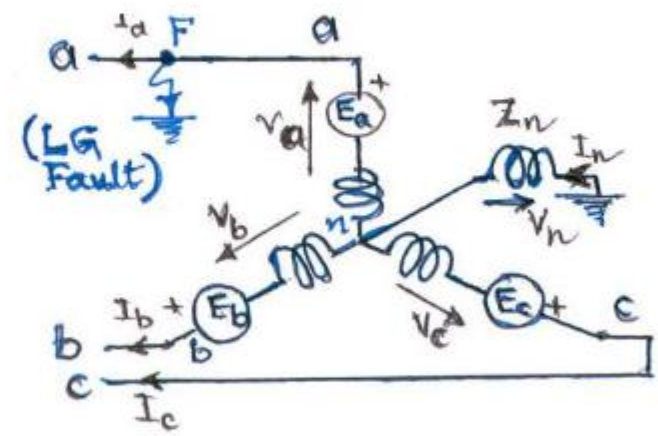


Figure 4.1 LG Fault on a Conventional Generator

A conventional generator is one that produces only the balanced voltages. Let Ea, nd Ec be the internally generated voltages and Zn be the neutral impedance. The fault is assumed to be on the phase'a' as shown in figure 4.1. Consider now the conditions under fault as under:

c.u.f.:

$$I_b = 0; \quad I_c = 0; \quad \text{and} \quad V_a = 0. \tag{4.3}$$

Now consider the symmetrical components of the current I_a with $I_b=I_c=0$, given by:

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} I_a \\ 0 \\ 0 \end{vmatrix} \tag{4.4}$$

Solving (4.4) we get,

$$I_{a1} = I_{a2} = I_{a0} = (I_a/3) \tag{4.5}$$

Further, using equation (4.5) in (4.2), we get,

$$\begin{vmatrix} V_{a0} \\ V_{a1} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \end{vmatrix} \begin{vmatrix} I_{a1} \\ I_{a1} \end{vmatrix}$$

$$V_{a2} \quad 0 \qquad 0 \quad 0 \quad Z_2 \quad I_{a1} \tag{4.6}$$

Pre-multiplying equation (4.6) throughout by [1 1 1], we get,

$$V_{a1}+V_{a2}+V_{a0} = - I_{a1}Z_0 + E_a - I_{a1}Z_1 - I_{a2}Z_2$$

i.e., $Va = E_a - I_{a1} (Z_1 + Z_2 + Z_0) = \text{zero},$

Or in other words,

$$\mathbf{I_{a1} = [E_a/(Z_1 + Z_2 + Z_0)]} \tag{4.7}$$

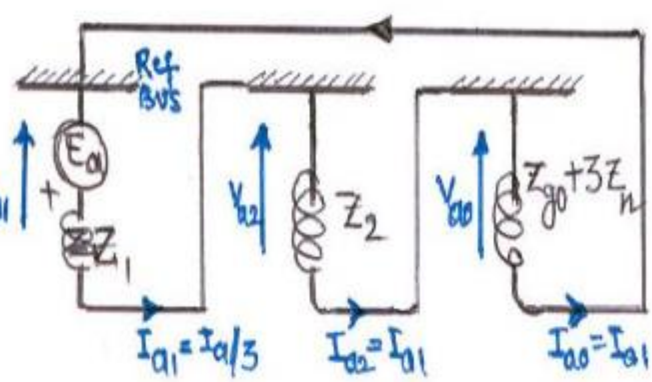


Figure 4.2 Connection of sequence networks for LG Fault on phase a of a Conventional Generator

The equation (4.7) derived as above implies that the three sequence networks are connected in series to simulate a LG fault, as shown in figure 4.2. Further we have the following relations satisfied under the fault conditions:

1. $I_{a1} = I_{a2} = I_{a0} = (I_a/3) = [E_a/(Z_1 + Z_2 + Z_0)]$
2. **Fault current** $\mathbf{I_f = I_a = 3I_{a1} = [3E_a/(Z_1 + Z_2 + Z_0)]}$
3. $V_{a1} = E_a - I_{a1}Z_1 = E_a(Z_2+Z_0)/(Z_1+Z_2+Z_0)$
4. $V_{a2} = - E_aZ_2/(Z_1+Z_2+Z_0)$
5. $V_{a0} = - E_aZ_0/(Z_1+Z_2+Z_0)$
6. Fault phase voltage $V_a = 0$,
7. Sound phase voltages $V_b = a^2V_{a1}+aV_{a2}+V_{a0}$; $V_c = aV_{a1}+a^2V_{a2}+V_{a0}$
8. Fault phase power: $V_aI_a^* = 0$, Sound pahse powers: $V_bI_b^* = 0$, and $V_cI_c^* = 0$,
9. If $Z_n = 0$, then $Z_0 = Z_{g0}$.

10. If $Z_n = \infty$, then $Z_0 = \infty$, i.e., the zero sequence network is open so that then, $I_f=I_a=0$.

4.3 LINE TO LINE FAULT ON A CONVENTIONAL GENERATOR

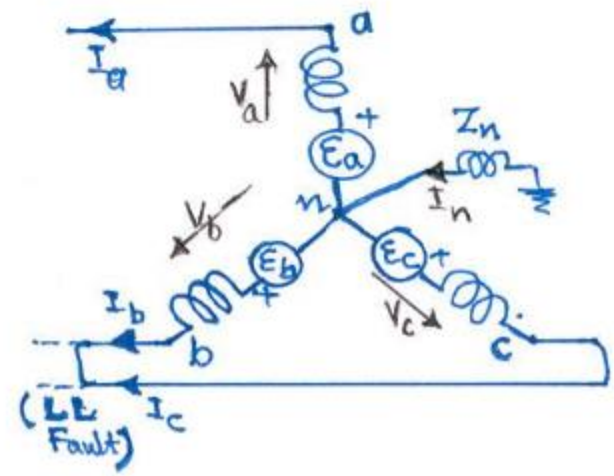


Figure 4.3 LL Fault on a Conventional Generator

Consider a line to line fault between phase 'b' and phase 'c' as shown in figure 4.3, at the terminals of a conventional generator, whose neutral is grounded through a reactance. Consider now the conditions under fault as under:

c.u.f.:

$$I_a = 0;\ I_b = -I_c;\ \text{and}\ V_b = V_c \tag{4.8}$$

Now consider the symmetrical components of the voltage V_a with $V_b=V_c$, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} V_a \\ V_b \\ V_b \end{vmatrix} \tag{4.9}$$

Solving (4.4) we get,

$$V_{a1} = V_{a2} \tag{4.10}$$

Further, consider the symmetrical components of current I_a with $I_b=-I_c$, and $I_a=0$; given by:

$$\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} 0 \\ I_b \\ -I_b \end{vmatrix} \tag{4.11}$$

Solving (4.11) we get,

$$I_{a0} = 0;\ \text{and } I_{a2} = -I_{a1} \tag{4.12}$$

Using equation (4.10) and (4.12) in (4.2), and since $V_{a0} = 0$ (I_{a0} being 0), we get,

$$\begin{vmatrix} 0 \\ V_{a1} \\ V_{a1} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} 0 \\ I_{a1} \\ -I_{a1} \end{vmatrix} \tag{4.13}$$

Pre-multiplying equation (4.13) throughout by [0 1 -1], we get,

$$V_{a1} - V_{a1} = E_a - I_{a1}Z_1 - I_{a1}Z_2 = 0$$

Or in other words,

$$\mathbf{I_{a1} = [E_a/(Z_1 + Z_2)]} \tag{4.14}$$

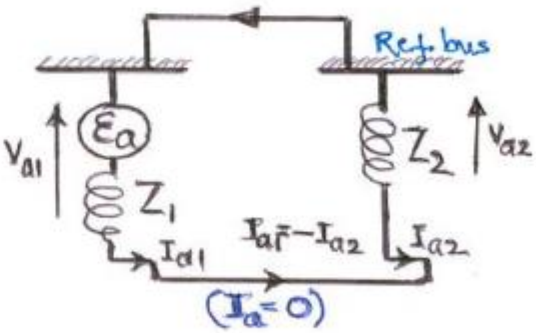


Figure 4.4 Connection of sequence networks for LL Fault on phases b & c of a Conventional Generator

The equation (4.14) derived as above implies that the three sequence networks are connected such that the zero sequence network is absent and only the positive and negative sequence networks are connected in series-opposition to simulate the LL fault, as shown in figure 4.4. Further we have the following relations satisfied under the fault conditions:

1. $I_{a1} = - I_{a2} = [E_a/(Z_1 + Z_2)]$ and $I_{a0} = 0$,
2. Fault current $\mathbf{I_f = I_b = - I_c = [\sqrt{3}E_a/(Z_1 + Z_2)]}$ (since $I_b = (a^2\text{-}a)I_{a1} = \sqrt{3}I_{a1}$)
3. $V_{a1} = E_a - I_{a1}Z_1 = E_aZ_2/(Z_1+Z_2)$
4. $V_{a2} = V_{a1} = E_aZ_2/(Z_1+Z_2)$
5. $V_{a0} = 0$,
6. Fault phase voltages; $V_b = V_c = aV_{a1}+a^2V_{a2}+V_{a0} = (a+a^2)V_{a1} = - V_{a1}$
7. Sound phase voltage; $V_a = V_{a1}+V_{a2}+V_{a0} = 2V_{a1}$;
8. Fault phase powers are $V_bI_b^*$ and $V_cI_c^*$,
9. Sound phase power: $V_aI_a^* = 0$,

10. Since $I_{a0}=0$, the presence of absence of neutral impedance does not make any difference in the analysis.

4.4 DOUBLE LINE TO GROUND FAULT ON A CONVENTIONAL GENERATOR

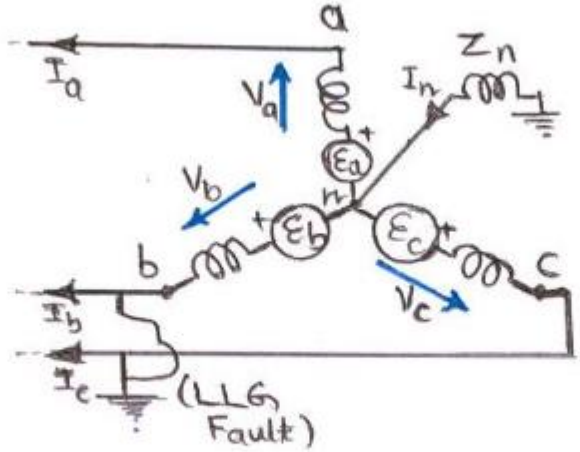


Figure 4.5 LLG Fault on a Conventional Generator

Consider a double-line to ground fault at the terminals of a conventional unloaded generator, whose neutral is grounded through a reactance, between phase 'b' and phase 'c' as shown in figure 4.5, Consider now the conditions under fault as under:

c.u.f.:

$$I_a = 0 \text{ and } V_b = V_c = 0 \qquad (4.15)$$

Now consider the symmetrical components of the voltage with $V_b=V_c=0$, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} V_a \\ 0 \\ 0 \end{vmatrix} \qquad (4.16)$$

Solving (4) we get,

$$V_{a1} = V_{a2} = V_{a0} = V_a/3 \qquad (4.17)$$

Consider now the sequence equations (4.2) as under,

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \qquad (4.18)$$

Pre-multiplying equation (4.18) throughout by

$$Z^{-1} = \begin{matrix} 1/Z_0 & 0 & 0 \\ 0 & 1/Z_1 & 0 \\ 0 & 0 & 1/Z_2 \end{matrix} \qquad (4.19)$$

We get,

$$Z^{-1}\begin{vmatrix} V_{a1} \\ V_{a1} \\ V_{a1} \end{vmatrix} = Z^{-1}\begin{vmatrix} 0 \\ E_a \\ 0 \end{vmatrix} - Z^{-1}\begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix}\begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \qquad (4.20)$$

Using the identity: $V_{a1} = (E_a - I_{a1}Z_1)$ in equation (4.19), pre-multiplying throughout by [1 1 1] and finally adding, we get,

$$E_a/Z_0 - I_{a1}(Z1/Z_0) + (E_a/Z_1) - I_{a1} + E_a/Z_2 - I_{a1}(Z1/Z_2) = (E_a/Z_1) - (I_{a0}+I_{a1}+I_{a2})$$

$$= (E_a/Z_1) - I_a \quad = (E_a/Z_1) \qquad (4.21)$$

Since $I_a = 0$, solving the equation (4.21), we get,

$$\mathbf{I_{a1} = \{ E_a / [Z_1 + Z_2Z_0/(Z_2+Z_0)] \}} \qquad (4.22)$$

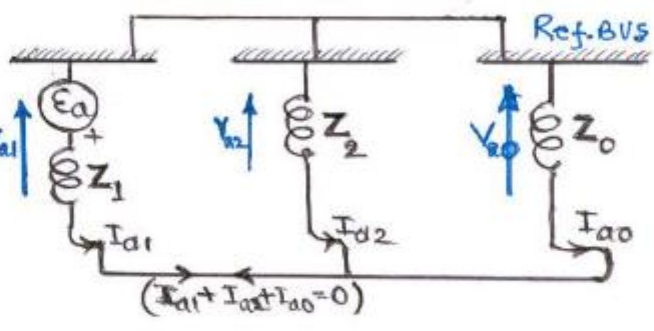


Figure4.6 Connection of sequence networks for LLG Fault on phases b and c of a Conventional Generator

The equation (4.22) derived as above implies that, to simulate the LLG fault, the three sequence networks are connected such that the positive network is connected in series with the parallel combination of the negative and zero sequence networks, as shown in figure 4.6. Further we have the following relations satisfied under the fault conditions:

1. $I_{a1} = \{E_a/ [Z_1+Z_2Z_0/(Z_2+Z_0)]\}$; $I_{a2} = -I_{a1}Z_0/(Z_2 + Z_0)$ and $I_{a0} = -I_{a1}Z_2/(Z_2 + Z_0)$,
2. Fault current I_f: $I_{a0}=(1/3)(I_a+I_b+I_c) = (1/3)(I_b+I_c) = I_f/3$, Hence $\mathbf{I_f = 3I_{a0}}$
3. $I_a = 0$, $V_b=V_c=0$ and hence $V_{a1}=V_{a2}=V_{a0}=V_a/3$
4. Fault phase voltages; $V_b = V_c = 0$
5. Sound phase voltage; $Va = V_{a1}+V_{a2}+V_{a0} = 3V_{a1}$;
6. Fault phase powers are $V_bI_b^* = 0$, and $V_cI_c^* = 0$, since $V_b=V_c=0$

7. Healthy phase power: $V_a I_a^* = 0$, since $I_a=0$
8. If $Z_0=\infty$, (i.e., the ground is isolated), then $I_{a0}=0$, and hence the result is the same as that of the LL fault [with $Z_0=\infty$, equation (4.22) yields equation (4.14)].

4.5 THREE PHASE TO GROUND FAULT ON A CONVENTIONAL GENERATOR

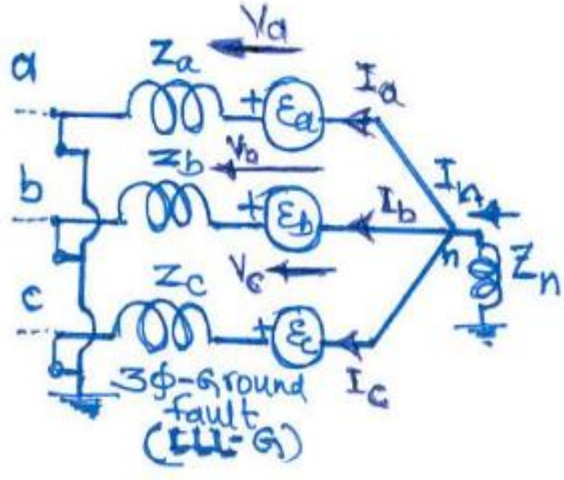


Figure 4.7 Three phase ground Fault on a Conventional Generator

Consider a three phase to ground (LLLG) fault at the terminals of a conventional unloaded generator, whose neutral is grounded through a reactance, between all its three phases a, b and c, as shown in figure 4.7, Consider now the conditions under fault as under:

c.u.f.:

$$V_a = V_b = V_c = 0, \; I_a + I_b + I_c = 0 \tag{4.23}$$

Now consider the symmetrical components of the voltage with $V_a=V_b=V_c= 0$, given by:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = (1/3) \begin{vmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{vmatrix} \begin{vmatrix} 0 \\ 0 \\ 0 \end{vmatrix} \tag{4.24}$$

Solving (4.24) we get,

$$V_{a1} = V_{a2} = V_{a0} = 0 \tag{4.25}$$

Thus we have

$$V_{a1} = E_{a1} - I_{a1}Z_1 \tag{4.26}$$

So that after solving for I_{a1} we, get,

$$\mathbf{I_{a1} = [\ E_a / Z_1\ ]} \tag{4.27}$$

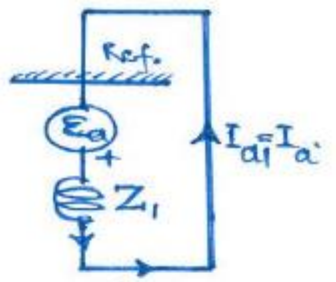


Figure 4.8 Connection of sequence networks for 3-phase ground Fault on phases b and c of a Conventional Generator

The equation (4.26) derived as above implies that, to simulate the 3-phase ground fault, the three sequence networks are connected such that the negative and zero sequence networks are absent and only the positive sequence network is present, as shown in figure 4.8. Further the fault current, I_f in case of a 3-phase ground fault is given by

$$\mathbf{I_f = I_{a1} = I_a = (E_a/Z_1)} \tag{4.28}$$

It is to be noted that the presence of a neutral connection without or with a neutral impedance, Z_n will not alter the simulated conditions in case of a three phase to ground fault.

4.6 UNSYMMETRICAL FAULTS ON POWER SYSTEMS

In all the analysis so far, only the fault at the terminals of an unloaded generator have been considered. However, faults can also occur at any part of the system and hence the power system fault at any general point is also quite important. The analysis of unsymmetrical fault on power systems is done in a similar way as that followed thus far for the case of a fault at the terminals of a generator. Here, instead of the sequence impedances of the generator, each and every element is to be replaced by their corresponding sequence impedances and the fault is analyzed by suitably connecting them together to arrive at the Thevenin equivalent impedance if that given sequence. Also, the internal voltage of the generators of the equivalent circuit for the positive sequence network is now V_f (and not E_a), the pre-fault voltage to neutral at the point of fault (PoF) (ref. Figure 4.9).

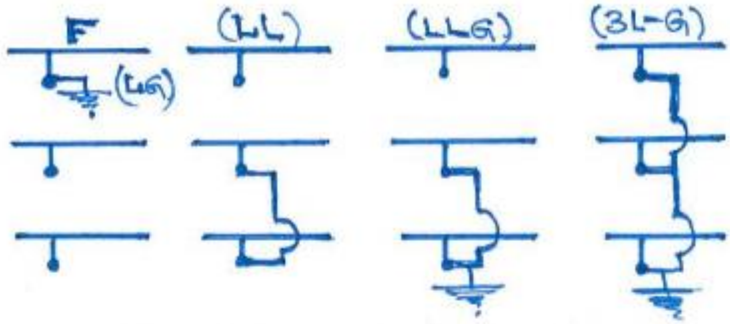


Figure 4.9 Unsymmetrical faults in Power Systems

Thus, for all the cases of unsymmetrical fault analysis considered above, the sequence equations are to be changed as under so as to account for these changes:

$$\begin{vmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{vmatrix} = \begin{vmatrix} 0 \\ V_f \\ 0 \end{vmatrix} - \begin{vmatrix} Z_0 & 0 & 0 \\ 0 & Z_1 & 0 \\ 0 & 0 & Z_2 \end{vmatrix} \begin{vmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{vmatrix} \tag{4.29}$$

(i) LG Fault at any point F of a given Power system

Let phase 'a' be on fault at F so that then, the c.u.f. would be:

$I_b = 0;\ I_c = 0;$ and $V_a = 0.$

Hence the derived conditions under fault would be:

$$I_{a1} = I_{a2} = I_{a0} = (I_a/3)$$
$$I_{a1} = [V_f / (Z_1 + Z_2 + Z_0)] \text{ and}$$
$$I_f = 3I_{a1} \tag{4.30}$$

(ii) LL Fault at any point F of a given Power system

Let phases 'b' and 'c' be on fault at F so that then, the c.u.f. would be:

$I_a = 0;\ I_b = - I_c;$ and $V_b = V_c$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2};\ I_{a0} = 0;\ I_{a2} = -I_{a1}$$
$$I_{a1} = [V_f / (Z_1 + Z_2)] \text{ and}$$
$$I_f = I_b = - I_c = [\sqrt{3}\, V_f / (Z_1 + Z_2)] \tag{4.31}$$

(ii) LLG Fault at any point F of a given Power system

Let phases 'b' and 'c' be on fault at F so that then, the c.u.f. would be:

$I_a = 0$ and $V_b = V_c = 0$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2} = V_{a0} = (V_a/3)$$

$$I_{a1} = \{V_f / [Z_1+Z_2Z_0/(Z_2+Z_0)]\}$$

$$I_{a2} = -I_{a1}Z_0/(Z_2 + Z_2);\ I_{a0} = -I_{a1}Z_2/(Z_2 + Z_2) \text{ and}$$

$$\mathbf{I_f = 3I_{a0}} \qquad (4.32)$$

(ii) Three Phase Fault at any point F of a given Power system

Let all the 3 phases a, b and c be on fault at F so that then, the c.u.f. would be:

$$V_a = V_b = V_c = 0,\ I_a + I_b + I_c = 0$$

Hence the derived conditions under fault would be:

$$V_{a1} = V_{a2} = V_{a0} = V_a/3$$

$$V_{a0} = V_{a1} = V_{a2} = 0;\ I_{a0} = I_{a2} = 0,$$

$$I_{a1} = [V_f/Z_1] \text{ and } I_f = I_{a1} = I_a \qquad (4.33)$$

4.7 OPEN CONDUCTOR FAULTS

Various types of power system faults occur in power systems such as the *shunt type faults* (LG, LL, LLG, LLLG faults) and *series type faults* (open conductor and cross country faults). While the symmetrical fault analysis is useful in determination of the rupturing capacity of a given protective circuit breaker, the unsymmetrical fault analysis is useful in the determination of relay setting, single phase switching and system stability studies.

When one or two of a three-phase circuit is open due to accidents, storms, etc., then unbalance is created and the asymmetrical currents flow. Such types of faults that come in series with the lines are refered as the *open conductor faults*. The open conductor faults can be analyzed by using the sequence networks drawn for the system under consideration as seen from the point of fault, F. These networks are then suitably connected to simulate the given type of fault. The following are the cases required to be analyzed (ref. fig.4.10).

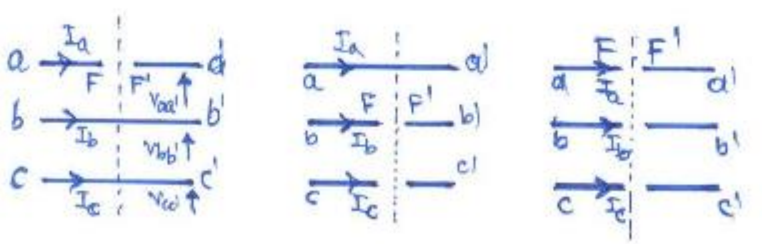


Figure 4.10 Open conductor faults.

(i) Single Conductor Open Fault: consider the phase 'a' conductor open so that then the conditions under fault are:

$$I_a = 0;\ V_{bb'} = V_{cc'} = 0$$

The derived conditions are:

$I_{a1} + I_{a2} + I_{a0} = 0$ and

$$V_{aa1'} = V_{aa2'} = V_{aa0'} = (V_{aa'}/3) \tag{4.34}$$

These relations suggest a parallel combination of the three sequence networks as shown in fig. 4.11.

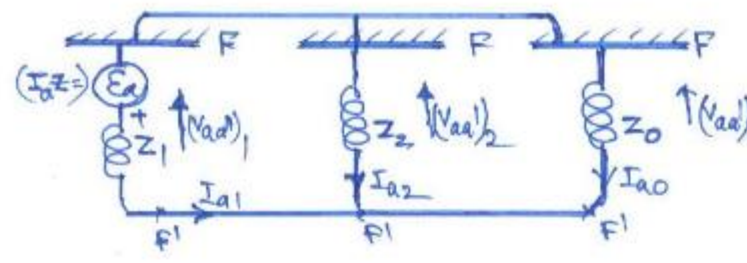


Figure 4.11 Sequence network connection for 1-conductor open fault

It is observed that a single conductor fault is similar to a LLG fault at the fault point F of the system considered.

(ii) Two Conductor Open Fault: consider the phases 'b' and 'c' under open condition so that then the conditions under fault are:

$I_b = I_c = 0;\ V_{aa'} = 0$

The derived conditions are:

$I_{a1} = I_{a2} = I_{a0} = I_a/3$ and

$$V_{aa1'} = V_{aa2'} = V_{aa0'} = 0 \tag{4.35}$$

These relations suggest a series combination of the three sequence networks as shown in fig. 4.12. It is observed that a double conductor fault is similar to a LG fault at the fault point F of the system considered.

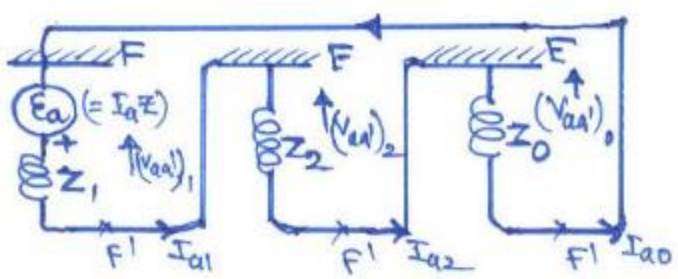


Figure 4.12 Sequence network connection for 2-conductor open fault.

(iii) Three Conductor Open Fault: consider all the three phases a, b and c, of a 3-phase system conductors be open. The conditions under fault are:

$I_a + I_b + I_c = 0$

The derived conditions are:

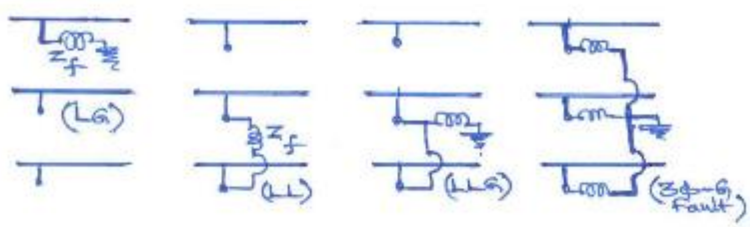
$I_{a1} = I_{a2} = I_{a0} = 0$ and

$V_{a0} = V_{a2} = 0$ and $V_{a1} = V_f$ (4.36)

These relations imply that the sequence networks are all open circuited. Hence, in a strict analytical sense, this is not a fault at all!

4.8 FAULTS THROUGH IMPEDANCE

All the faults considered so far have comprised of a direct short circuit from one or two lines to ground. The effect of impedance in the fault is found out by deriving equations similar to those for faults through zero valued neutral impedance. The connections of the hypothetical stubs for consideration of faults through fault impedance Z_f are as shown in figure 4.13.



Fig

ure 4.13 Stubs Connections for faults through fault impedance Z_f.

(i) LG Fault at any point F of a given Power system through Z_f

Let phase 'a' be on fault at F *through* Z_f, so that then, the c.u.f. would be:

$I_b = 0$; $I_c = 0$; and $V_a = 0$.

Hence the derived conditions under fault would be:

$I_{a1} = I_{a2} = I_{a0} = (I_a/3)$

$I_{a1} = [V_f / (Z_1 + Z_2 + Z_0 + 3Z_f)]$ and

$I_f = 3I_{a1}$ (4.37)

(ii) LL Fault at any point F of a given Power system through Z_f

Let phases 'b' and 'c' be on fault at F *through* Z_f, so that then, the c.u.f. would be:

$I_a = 0$; $I_b = - I_c$; and $V_b = V_c$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2}$; $I_{a0} = 0$; $I_{a2} = -I_{a1}$

$I_{a1} = [V_f / (Z_1 + Z_2 + Z_f)]$ and

$I_f = I_b = - I_c = [\sqrt{3}\, V_f / (Z_1 + Z_2 + Z_f)]$ (4.38)

(iii) LLG Fault at any point F of a given Power system through Z_f

Let phases 'b' and 'c' be on fault at F *through* Z_f, so that then, the c.u.f. would be:

$I_a = 0$ and $V_b = V_c = 0$

Hence the derived conditions under fault would be:

$V_{a1} = V_{a2} = V_{a0} = (V_a/3)$

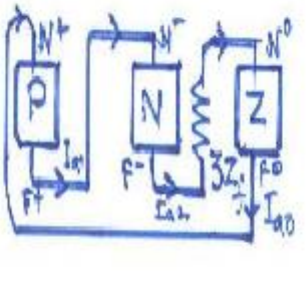
$I_{a1} = \{V_f / [Z_1+Z_2(Z_0+3Z_f)/(Z_2+Z_0+3Zf)]\}$

$I_{a2} = -I_{a1}(Z_0+3Z_f)/(Z_2+Z_0+3Z_f)$; $I_{a0} = -I_{a1}Z_2/(Z_2+(Z_0+3Z_f)$ and

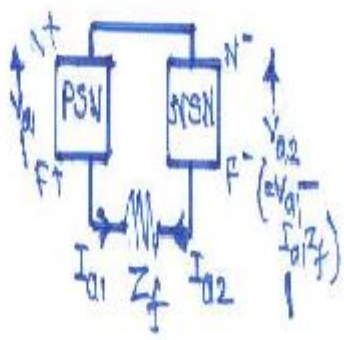
$\mathbf{I_f = 3I_{a0}}$ (4.39)

(iv) Three Phase Fault at any point F of a given Power system through Z_f

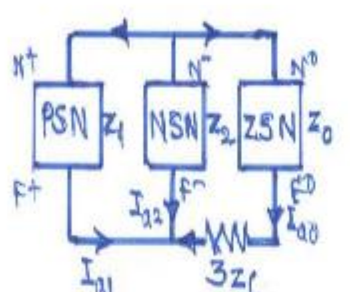
Let all the 3 phases a, b and c be on fault at F, through Z_f so that the c.u.f. would be: $V_a = I_aZ_f$; Hence the derived conditions under fault would be: $I_{a1} = [V_f/(Z_1+Z_f)$; The connections of the sequence networks for all the above types of faults through Z_f are as shown in figure 4.14.



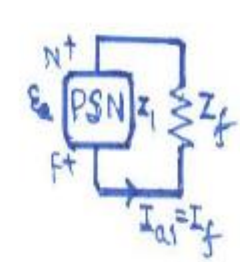
LG Fault



LL Fault



LLG Fault



3-Ph. Fault

Figure 4.15 Sequence network connections for faults through impedance

4.9 EXAMPLES

Example-1: A three phase generator with constant terminal voltages gives the following currents when under fault: 1400 A for a line-to-line fault and 2200 A for a line-to-ground fault. If the positive sequence generated voltage to neutral is 2 ohms, find the reactances of the negative and zero sequence currents.

Solution: Case a) Consider the conditions w.r.t. the LL fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2)]$$

$$I_f = I_b = - I_c = \sqrt{3}\, I_{a1}$$

$$= \sqrt{3}\, E_{a1} / (Z_1 + Z_2) \quad \text{or}$$

$$(Z_1 + Z_2) = \sqrt{3}\, E_{a1} / I_f$$

i.e., $2 + Z_2 = \sqrt{3}\, [2000/1400]$

Solving, we get, $Z_2 = 0.474$ ohms.

Case b) Consider the conditions w.r.t. a LG fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2 + Z_0)]$$

$$I_f = 3\, I_{a1}$$

$$= 3\, E_{a1} / (Z_1 + Z_2 + Z_0) \quad \text{or}$$

$$(Z_1 + Z_2 + Z_0) = 3\, E_{a1} / I_f$$

i.e., $2 + 0.474 + Z_0 = 3\, [2000/2200]$

Solving, we get, $Z_0 = 0.253$ ohms.

Example-2: A dead fault occurs on one conductor of a 3-conductor cable supplied y a 10 MVA alternator with earhed neutral. The alternator has +ve, -ve and 0-sequence components of impedances per phase respectively as: (0.5+j4.7), (0.2+j0.6) and (j0.43) ohms. The corresponding LN values for the cable up to the point of fault are: (0.36+j0.25), (0.36+j0.25) and (2.9+j0.95) ohms respectively. If the generator voltage at no load (E_{a1}) is 6600 volts between the lines, determine the (i)Fault current, (ii)Sequence components of currents in lines and (iii)Voltages of healthy phases.

Solution: There is LG fault on any one of the conductors. Consider the LG fault to be on conductor in phase a. Thus the fault current is given by:

(i) Fault current: $I_f = 3I_{a0} = [3E_a/(Z_1 + Z_2 + Z_0)]$

$$= 3(6600/\sqrt{3}) / (4.32+j7.18)$$

$$= 1364.24\ \angle 58.97^0.$$

(ii) Sequence components of line currents:

$I_{a1} = I_{a2} = I_{a0} = I_a/3 = I_f/3 = 454.75 \angle 58.97^0$.

(iii) Sound phase voltages:

$V_{a1} = E_a - I_{a1}Z_1 = E_a(Z_2+Z_0)/(Z_1+Z_2+Z_0) = 1871.83 \angle -26.17^0$,

$V_{a2} = - E_aZ_2/(Z_1+Z_2+Z_0) = 462.91 \angle 177.6^0$,

$V_{a0} = - E_aZ_0/(Z_1+Z_2+Z_0) = 1460.54 \angle 146.5^0$,

Thus,

Sound phase voltages $V_b = a^2V_{a1}+aV_{a2}+V_{a0} = 2638.73 \angle -165.8^0$ Volts,

And $V_c = aV_{a1}+a^2V_{a2}+V_{a0} = 3236.35 \angle 110.8^0$ Volts.

Example-3: A generator rated 11 kV, 20 MVA has reactances of X_1=15%, X_2=10% and X_0=20%. Find the reactances in ohms that are required to limit the fault current to 2 p.u. when a a line to ground fault occurs. Repeat the analysis for a LLG fault also for a fault current of 2 pu.

Solution: **Case a:** Consider the fault current expression for LG fault given by:

$I_f = 3\, I_{a0}$

i.e., $2.0 = 3Ea / j[X_1+X_2+X_0]$

$= 3(1.0\angle 0^0) / j[0.15+0.1+0.2+3X_n]$

Solving we get

$3X_n = 2.1$ pu

$= 2.1\ (Z_b)$ ohms $= 2.1\ (11^2/20) = 2.1(6.05)$

$= 12.715$ ohms.

Thus $X_n = 4.235$ ohms.

Case b: Consider the fault current expression for LLG fault given by:

$\mathbf{I_f = 3I_{a0} = 3}\ \{ -I_{a1}X_2/(X_2 + X_0+3X_n)\}= 2.0$,

where, $I_{a1} = \{E_a/ [X_1+X2(X0+3X_n)/(X_2+X_0+3X_n)]\}$

Substituting and solving for X_n we get,

$Xn = 0.078$ pu

$= 0.47$ ohms.

Example-4: A three phase 50 MVA, 11 kV generator is subjected to the various faults and the surrents so obtained in each fault are: 2000 A for a three phase fault; 1800 A for a line-to-line fault and 2200 A for a line-to-ground fault. Find the sequence impedances of the generator.

Solution: Case a) Consider the conditions w.r.t. the three phase fault:

$I_f = I_a = I_{a1} = E_{a1}/Z_1$

i.e., $2000 = 11000/ (\sqrt{3}Z_1)$

Solving, we get, $Z_1 = 3.18$ ohms (1.3 pu, with $Z_b = (11^2/50) = 2.42$ ohms).

Case b) Consider the conditions w.r.t. the LL fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2)]$$

$$I_f = I_b = - I_c = \sqrt{3}\, I_{a1}$$

$$= \sqrt{3}\, E_{a1} / (Z_1 + Z_2) \quad \text{or}$$

$$(Z_1 + Z_2) = \sqrt{3}\, E_{a1} / I_f$$

i.e., $3.18 + Z_2 = \sqrt{3}\ (11000/\sqrt{3})/1800$

Solving, we get, $Z_2 = 2.936$ ohms $= 1.213$ pu.

Case c) Consider the conditions w.r.t. a LG fault:

$$I_{a1} = [E_{a1}/(Z_1 + Z_2+Z_0)]$$

$$I_f = 3\, I_{a1}$$

$$= 3\, E_{a1} / (Z_1 + Z_2+Z_0) \quad \text{or}$$

$$(Z_1 + Z_2+Z_0) = 3\, E_{a1} / I_f$$

i.e., $3.18+ 2.936 + Z_0 = 3\ (11000/\sqrt{3})/ 2200$

Solving, we get, $Z_0 = 2.55$ ohms $= 1.054$ pu.

CHAPTER 5: POWER SYSTEM STABILITY

5.1 INTRODUCTION

Power system stability of modern large inter-connected systems is a major problem for secure operation of the system. Recent major black-outs across the globe caused by system instability, even in very sophisticated and secure systems, illustrate the problems facing secure operation of power systems. Earlier, stability was defined as the ability of a system to return to normal or stable operation after having been subjected to some form of disturbance. This fundamentally refers to the ability of the system to remain in synchronism. However, modern power systems operate under complex interconnections, controls and extremely stressed conditions. Further, with increased automation and use of electronic equipment, the quality of power has gained utmost importance, shifting focus on to concepts of voltage stability, frequency stability, inter-area oscillations etc.

The IEEE/CIGRE Joint Task Force on stability terms and conditions have proposed the following definition in 2004: "*Power System stability is the ability of an electric power system, for a given initial operating condition, to regain a state of operating equilibrium after being subjected to a physical disturbance, with most system variables bounded, so that practically the entire system remains intact*".

The Power System is an extremely non-linear and dynamic system, with operating parameters continuously varying. Stability is hence, a function of the initial operating condition and the nature of the disturbance. Power systems are continually subjected to small disturbances in the form of load changes. The system must be in a position to be able to adjust to the changing conditions and operate satisfactorily. The system must also withstand large disturbances, which may even cause structural changes due to isolation of some faulted elements.

A power system may be stable for a particular (large) disturbance and unstable for another disturbance. It is impossible to design a system which is stable under all disturbances. The power system is generally designed to be stable under those disturbances which have a high degree of occurrence. The response to a disturbance is extremely complex and involves practically all the equipment of the power system. For example, a short circuit leading to a line isolation by circuit breakers will cause variations in the power flows, network bus voltages and generators rotor speeds. The voltage variations will actuate the voltage regulators in the system and generator speed variations will actuate the prime mover governors; voltage and frequency variations will affect the system loads. In stable systems, practically all generators and loads remain connected, even though parts of the system may be isolated to preserve bulk operations. On the other hand, an unstable system condition could lead to cascading outages and a shutdown of a major portion of the power system.

5.2 CLASSIFICATION OF POWER SYSTEM STABILITY

The high complexity of stability problems has led to a meaningful classification of the power system stability into various categories. The classification takes into account the main system variable in which instability can be observed, the size of the disturbance and the time span to be considered for assessing stability.

5.2.1 ROTOR ANGLE STABILITY

Rotor angle stability refers to the ability of the synchronous machines of an interconnected power system to remain in synchronism after being subjected to a disturbance. Instability results in some generators accelerating (decelerating) and losing synchronism with other generators. Rotor angle stability depends on the ability of each synchronous machine to maintain equilibrium between electromagnetic torque and mechanical torque. Under steady state, there is equilibrium between the input mechanical torque and output electromagnetic torque of each generator, and its speed remains a constant. Under a disturbance, this equilibrium is upset and the generators accelerate/decelerate according to the mechanics of a rotating body. Rotor angle stability is further categorized as follows:

Small single (or small disturbance) rotor angle stability: It is the ability of the power system to maintain synchronism under small disturbances. In this case, the system equation can be linearized around the initial operating point and the stability depends only on the operating point and not on the disturbance. Instability may result in

(i) A non oscillatory or a periodic increase of rotor angle

(ii) Increasing amplitude of rotor oscillations due to insufficient damping.

The first form of instability is largely eliminated by modern fast acting voltage regulators and the second form of instability is more common. The time frame of small signal stability is of the order of 10-20 seconds after a disturbance.

Large-signal rotor angle stability or transient stability: This refers to the ability of the power system to maintain synchronism under large disturbances, such as short circuit, line outages etc. The system response involves large excursions of the generator rotor angles. Transient stability depends on both the initial operating point and the disturbance parameters like location, type, magnitude etc. Instability is normally in the form of a periodic angular separation. The time frame of interest is 3-5 seconds after disturbance.

The term dynamic stability was earlier used to denote the steady-state stability in the presence of automatic controls (especially excitation controls) as opposed to manual controls. Since all generators are equipped with automatic controllers today, dynamic stability has lost relevance and the Task Force has recommended against its usage.

5.2.2 VOLTAGE STABILITY

Voltage stability refers to the ability of a power system to maintain steady voltages at all buses in the system after being subjected to a disturbance. It depends on the ability of the system to maintain equilibrium between load demand and load supply. Instability results in a progressive fall or rise of voltages of some buses, which could lead to loss of load in an area or tripping of transmission lines, leading to cascading outages. This may eventually lead to loss of synchronism of some generators.

The cause of voltage instability is usually the loads. A run-down situation causing voltage instability occurs when load dynamics attempt to restore power consumption beyond the capability of the transmission network. Voltage stability is also threatened when a disturbance increases the reactive power demand beyond the sustainable capacity of the available reactive power resources. Voltage stability is categorized into the following sub-categories:

Small – disturbance voltage stability: It refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in load. This is primarily influenced by the load characteristics and the controls at a given point of time.

Large disturbance voltage stability: It refers to the systems ability to maintain steady voltages following large disturbances; It requires computation of the non-linear response of the power system to include interaction between various devices like motors, transformer tap changers and field current limiters. Short term voltage stability involves dynamics of fast acting load components and period of interest is in the order of several seconds. Long term voltage stability involves slower acting equipment like tap-changing transformers and generator current limiters. Instability is due to loss of long-term equilibrium.

5.2.3 FREQUENCY STABILITY

Frequency stability refers to the ability of a power system to maintain steady frequency following a severe disturbance, causing considerable imbalance between generation and load. Instability occurs in the form of sustained frequency swings leading to tripping of generating units or loads. During frequency swings, devices such as under frequency load shedding, generator controls and protection equipment get activated in a few seconds. However, devices such as prime mover energy supply systems and load voltage regulators respond after a few minutes. Hence, frequency stability could be a short-term or a long-term phenomenon.

5.3 MECHANICS OF ROTATORY MOTION

Since a synchronous machine is a rotating body, the laws of mechanics of rotating bodies are applicable to it. In rotation we first define the fundamental quantities. The angle θ_m is defined, with respect to a circular arc with its center at the vertex of the angle, as the ratio of the arc length *s* to radius *r*.

$$\theta_m = \frac{s}{r} \tag{5.1}$$

The unit is radian. Angular velocity ω_m is defined as

$$\omega_m = \frac{d\theta_m}{dt} \tag{5.2}$$

and angular acceleration as

$$\alpha = \frac{d\omega_m}{dt} = \frac{d^2\theta_m}{dt^2} \tag{5.3}$$

The torque on a body due to a tangential force F at a distance r from axis of rotation is given by $\quad T = r\,F \qquad (5.4)$

The total torque is the summation of infinitesimal forces, given by

$$T = \int r\,dF \tag{5.5}$$

The unit of torque is N-m. When torque is applied to a body, the body experiences angular acceleration. Each particle experiences a tangential acceleration $a = r\alpha$, where r is the distance of the particle from axis of rotation. The tangential force required to accelerate a particle of mass *dm* is

$$dF = a\,dm = r\,\alpha\,dm \tag{5.6}$$

The torque required for the particle is

$$dT = r\,dF = r^2\,\alpha\,dm \tag{5.7}$$

and that required for the whole body is given by

$$T = \alpha \int r^2 dm = I\,\alpha \tag{5.8}$$

Here, $\quad I = \int r^2 dm \qquad (5.9)$

It is called the moment of inertia of the body. The unit is Kg – m^2. If the torque is assumed to be the result of a number of tangential forces F, which act at different points of the body

$$T = \sum r\,F$$

Now each force acts through a distance, $ds = r\,d\theta_m$ and the work done is $\sum F \cdot ds$ i.e.,

$$dW = \sum F\, r\, d\theta_m - d\theta_m\, T$$

$$W = \int T\, d\theta_m \tag{5.10}$$

and
$$T = \frac{dW}{d\theta_m} \tag{5.11}$$

Thus the unit of torque may also be Joule per radian. The power is defined as rate of doing work. Using (5.11)

$$P = \frac{dW}{dt} = \frac{T\, d\theta_m}{dt} = T\,\omega_m \tag{5.12}$$

The angular momentum M is defined as

$$M - I\,\omega_m \tag{5.13}$$

And the kinetic energy is given by

$$KE = \frac{1}{2} I \omega_m^{\ 2} = \frac{1}{2} M\,\omega_m \tag{5.14}$$

From (5.14) we can see that the unit of M has to be J-sec/rad.

5.4 SWING EQUATION:

The laws of rotation developed in section.3 are applicable to the synchronous machine. From(.5.8)

$$I\alpha = T$$

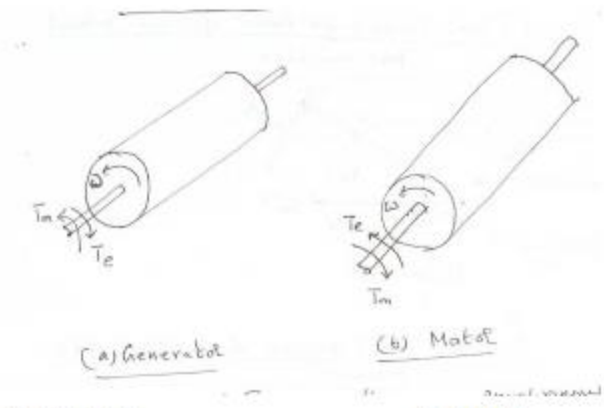
or
$$\frac{I\, d^2\theta_m}{dt^2} = T \tag{5.15}$$

Here T is the net torque of all torques acting on the machine, which includes the shaft torque (due to prime mover of a generator or load on a motor), torque due to rotational losses (friction, windage and core loss) and electromagnetic torque.

Let T_m = shaft torque or mechanical torque corrected for rotational losses

T_e = Electromagnetic or electrical torque

For a generator T_m tends to accelerate the rotor in positive direction of rotation as shown in Fig 5.1. It also shows the corresponding torque for a motor with respect to the direction of rotation.



(a) Generator **(b) Motor**

Fig. 5.1 Torque acting on a synchronous machine

The accelerating torque for a generator is given by:

$$T_a = T_m - T_e \tag{5.16}$$

Under steady-state operation of the generator, T_m is equal to T_e and the accelerating torque is zero. There is no acceleration or deceleration of the rotor masses and the machines run at a constant synchronous speed. In the stability analysis in the following sections, T_m is assumed to be a constant since the prime movers (steam turbines or hydro turbines) do no act during the short time period in which rotor dynamics are of interest in the stability studies.

Now (5.15) has to be solved to determine θ_m as a function of time. Since θ_m is measured with respect to a stationary reference axis on the stator, it is the measure of the absolute rotor angle and increases continuously with time even at constant synchronous speed. Since machine acceleration /deceleration is always measured relative to synchronous speed, the rotor angle is measured with respect to a synchronously rotating reference axis. Let

$$\delta_m = \theta_m - \omega_{sm} t \tag{5.17}$$

where ω_{sm} is the synchronous speed in mechanical rad/s and δ_m is the angular displacement in mechanical radians. Taking the derivative of (5.17) we get

$$\frac{d\delta_m}{dt} = \frac{d\theta_m}{dt} \square \omega_{sm}$$

$$\frac{d^2\delta_m}{dt^2} = \frac{d^2\theta_m}{dt^2} \tag{5.18}$$

Substituting (5.18) in (5.15) we get

$$I\frac{d^2\delta_m}{dt^2} = T_a = T_m \square T_e \text{ N-m} \tag{5.19}$$

Multiplying by ω_m on both sides we get

$$\omega_m I\frac{d^2\delta_m}{dt^2} = \omega_m (T_m \square T_e) \text{ N-m} \tag{5.20}$$

From (5.12) and (5.13), we can write

$$M\frac{d^2\delta_m}{dt^2} = P_m - P_a \qquad W \tag{5.21}$$

where M is the angular momentum, also called inertia constant, P_m is shaft power input less rotational losses, P_e is electrical power output corrected for losses and P_a is the acceleration power. M depends on the angular velocity ω_m, and hence is strictly not a constant, because ω_m deviates from the synchronous speed during and after a disturbance. However, under stable conditions ω_m does not vary considerably and M can be treated as a constant. (21) is called the *"Swing equation"*. The constant M depends on the rating of the machine and varies widely with the size and type of the machine. Another constant called H constant (also referred to as inertia constant) is defined as

$$H = \frac{\begin{matrix}\textit{stored kinetic energy in mega joules}\\ \textit{at sychronous speed}\end{matrix}}{\textit{Machine rating in MVA}} \; MJ/MVA \tag{5.22}$$

H falls within a narrow range and typical values are given in Table 5.1. If the rating of the machine is G MVA, from (5.22) the stored kinetic energy is GH Mega Joules. From (5.14)

$$\mathrm{GH} = \frac{1}{2} M\omega_{sm} \quad \mathrm{MJ} \tag{5.23}$$

or

$$\mathrm{M} = \frac{2GH}{\omega_{sm}} \quad \text{MJ-s/mech rad} \tag{5.24}$$

The swing equation (5.21) is written as

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{G} = \frac{P_m - P_e}{G} \tag{5.25}$$

In (5.25) δ_m is expressed in mechanical radians and ω_{sm} in mechanical radians per second (the subscript *m* indicates mechanical units). If δ and ω have consistent units (both are mechanical or electrical units) (5.25) can be written as

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.26}$$

Here ω_s is the synchronous speed in electrical rad/s $(\omega_s = \left(\frac{p}{2}\right)\omega_{sm})$ and P_a is acceleration power in per unit on same base as H. For a system with an electrical frequency f Hz, (5.26) becomes

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.27}$$

when δ is in electrical radians and

$$\frac{H}{180 f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad \text{pu} \tag{5.28}$$

when δ is in electrical degrees. Equations (5.27) and (5.28) also represent the swing equation. It can be seen that the swing equation is a second order differential equation which can be written as two first order differential equations:

$$\frac{2H}{\omega_s} \frac{d\omega}{dt} = P_m - P_e \quad \text{pu} \tag{5.29}$$

$$\frac{d\delta}{dt} = \omega - \omega_s \tag{5.30}$$

In which ω, ω_s and δ are in electrical units. In deriving the swing equation, damping has been neglected.

Table 5.1 H constants of synchronous machines

Type of machine		H (MJ/MVA)
Turbine generator condensing	1800 rpm	9 – 6
	3600 rpm	7 – 4
Non condensing	3600 rpm	4 – 3
Water wheel generator		
	Slow speed < 200 rpm	2 – 3
	High speed > 200 rpm	2 – 4
Synchronous condenser	Large	1.25 } 25% less for hydrogen cooled
	Small	1.0 } 25% less for hydrogen cooled
Synchronous motor with load varying		
from 1.0 to 5.0		2.0

In defining the inertia constant H, the MVA base used is the rating of the machine. In a multi machine system, swing equation has to be solved for each machine, in which case, a common MVA base for the system has to chosen. The constant H of each machine must be consistent with the system base.

Let G_{mach} = Machine MVA rating (base)

G_{system} = System MVA base

In (5.25), H is computed on the machine rating $G = G_{mach}$

Multiplying (5.25) by $\frac{G_{mach}}{G_{system}}$ on both sides we get

$$\left(\frac{G_{mach}}{G_{system}}\right)\frac{2H}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = \frac{P_m - P_e}{G_{mach}}\left(\frac{G_{mach}}{G_{system}}\right) \tag{5.31}$$

$$\frac{2H_{system}}{\omega_{sm}}\frac{d^2\delta_m}{dt^2} = P_m - P_e \quad \text{pu (on system base)}$$

where $H_{system} = H\frac{G_{mach}}{G_{system}}$ (5.32)

In the stability analysis of a multi machine system, computation is considerably reduced if the number of swing equations to be solved is reduced. Machines within a plant normally swing together after a disturbance. Such machines are called coherent machines and can be replaced by a single equivalent machine, whose dynamics reflects the dynamics of the plant. The concept is best understood by considering a two machine system.

5.4.1 SWING EQUATION OF TWO COHERENT MACHINES

The swing equations for two machines on a common system base are:

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1} \text{ pu} \tag{5.33}$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2} \text{ pu} \tag{5.34}$$

Now $\delta_1 = \delta_2 = \delta$ (since they swing together). Adding (5.33) and (5.34) we get

$$\frac{2H_{eq}}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \tag{5.35}$$

Where $H_{eq} = H_1 + H_2$

$$P_m = P_{m1} + P_{m2}$$

$$P_e = P_{e1} + P_{e2}$$

The relation (5.35) represents the dynamics of the single equivalent machine.

5.4.2 SWING EQUATION OF TWO NON – COHERENT MACHINES

For any two non – coherent machines also (5.33) and (5.34) are valid. Subtracting (5.34) from (33) we get

$$\frac{2}{\omega_s}\frac{d^2\delta_1}{dt^2} - \frac{2}{\omega_s}\frac{d^2\delta_2}{dt^2} = \frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2} \tag{5.36}$$

Multiplying both sides by $\dfrac{H_1 H_2}{H_1 + H_2}$ we get

$$\frac{2}{\omega_s}\left(\frac{H_1 H_2}{H_1+H_2}\right)\frac{d^2(\delta_1-\delta_2)}{dt^2}=\frac{P_{m1}H_2-P_{m2}H_1}{H_1+H_2}-\frac{P_{e1}H_2-p_{e2}H_1}{H_1+H_2}$$

i.e

$$\frac{2}{\omega_s}H_{12}\frac{d^2\delta_{12}}{dt^2}=P_{m12}-P_{e12} \tag{5.37}$$

where $\delta_{12}=\delta_1-\delta_2$, the relative angle of the two machines

$$H_{12}=\frac{H_1 H_2}{H_1+H_2}$$

$$P_{m12}=\frac{p_{m1}\,H_2-p_{m2}\,H_1}{H_1+H_2}$$

$$P_{e12}=\frac{p_{e1}\,H_2-p_{e2}\,H_1}{H_1+H_2}$$

From (5.37) it is obvious that the swing of a machine is associated with dynamics of other machines in the system. To be stable, the angular differences between all the machines must decrease after the disturbance. In many cases, when the system loses stability, the machines split into two coherent groups, swinging against each other. Each coherent group of machines can be replaced by a single equivalent machine and the relative swing of the two equivalent machines solved using an equation similar to (5.37), from which stability can be assessed.

The acceleration power is given by $P_a = P_m - P_e$. Hence, under the condition that P_m is a constant, an accelerating machine should have a power characteristic, which would increase P_e as δ increases.

This would reduce P_a and hence the acceleration and help in maintaining stability. If on the other hand, P_e decreases when δ increases, P_a would further increase which is detrimental to stability. Therefore, $\frac{\partial P}{\partial \delta}$ must be positive for a stable system. Thus the power-angle relationship plays a crucial role in stability.

5.5 POWER–ANGLE EQUATION:

In solving the swing equation, certain assumptions are normally made

(i) Mechanical power input P_m is a constant during the period of interest, immediately after the disturbance

(ii) Rotor speed changes are insignificant.

(iii) Effect of voltage regulating loop during the transient is neglected i.e the excitation is assumed to be a constant.

As discussed in section 4, the power–angle relationship plays a vital role in the solution of the swing equation.

5.5.1 POWER–ANGLE EQUATION FOR A NON–SALIENT POLE MACHINE:

The simplest model for the synchronous generator is that of a constant voltage behind an impedance. This model is called the classical model and can be used for cylindrical rotor (non–salient pole) machines. Practically all high–speed turbo alternators are of cylindrical rotor construction, where the physical air gap around the periphery of the rotor is uniform. This type of generator has approximately equal magnetic reluctance, regardless of the angular position of the rotor, with respect to the armature mmf. The phasor diagram of the voltages and currents at constant speed and excitation is shown in Fig. 5.2.

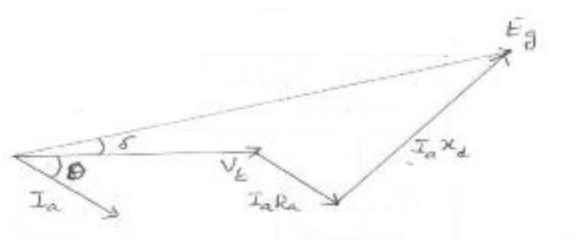


Fig 5.2 Phasor diagram of a non–salient pole synchronous generator

E_g = Generator internal emf.

V_t = Terminal voltage

θ = Power factor angle

I_a = Armature current

R_a = Armature resistance

x_d = Direct axis reactance

The power output of the generator is given by the real part of $E_g I_a^*$.

$$I_a = \frac{E_g \angle\delta - V_t \angle 0^\circ}{R_a + jx_d} \tag{5.38}$$

Neglecting R_a, $$I_a = \frac{E_g \angle\delta - V_t \angle 0^\circ}{j x_d}$$

$$P = \mathbf{R} \left\{ \left(E_g \angle\delta\right) \left(\frac{E_g \angle 90^\circ - \delta}{x_d} - \frac{V_t \angle 90^\circ}{x_d} \right)^* \right\}$$

$$= \frac{E_g^2 \cos 90^\circ}{x_d} - \frac{E_g V_t \cos\left(90^\circ + \delta\right)}{x_d}$$

$$= \frac{E_g V_t \sin\delta}{x_d} \tag{5.39}$$

(Note- **R** stands for real part of). The graphical representation of (9.39) is called the power angle curve and it is as shown in Fig 5.3.

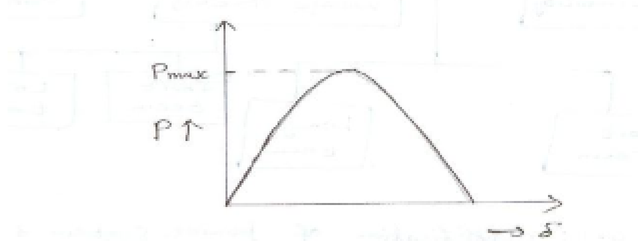


Fig 5.3 Power angle curve of a non – salient pole machine

The maximum power that can be transferred for a particular excitation is given by $\frac{E_g V_t}{x_d}$ at $\delta = 90^\circ$.

5.5.2 POWER ANGLE EQUATION FOR A SALIENT POLE MACHINE:

Here because of the salient poles, the reluctance of the magnetic circuit in which flows the flux produced by an armature mmf in line with the quadrature axis is higher than that of the magnetic circuit in which flows the flux produced by the armature mmf in line with the direct axis. These two components of armature mmf are proportional to the corresponding components of armature current. The component of armature current producing an mmf acting in line with direct axis is called the direct component, I_d. The component of armature current producing an mmf acting in line with the quadrature axis is called the quadrature axis component, I_q. The phasor diagram is shown in Fig 5, with same terminology as Fig 5.4 and R_a neglected.

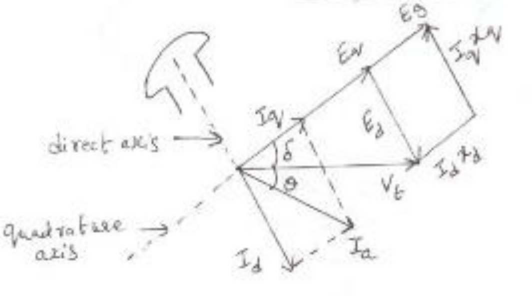


Fig 5.4 Phasor diagram of a salient pole machine

Power output $\quad P = V_t I_a \cos\theta$

$$= E_d I_d + E_q I_q \tag{5.40}$$

From Fig 5.4, $\quad E_d = V_t \sin\delta \;;\quad E_q = V_t \cos\delta$

$$I_d = \frac{E_g - E_q}{x_d} = I_a \sin(\delta + \theta)$$

$$I_q = \frac{E_d}{x_q} = I_a \cos(\delta + \theta) \tag{5.41}$$

Substituting (5.41) in (5.40), we obtain

$$P = \frac{E_g V_t \sin\delta}{x_d} + \frac{V_t^2 (x_d - x_q) \sin 2\delta}{2 x_d x_q} \tag{5.42}$$

the relation (5.42) gives the steady state power angle relationship for a salient pole machine. The second term does not depend on the excitation and is called the reluctance power component. This component makes the maximum power greater than in the classical model. However, the angle at which the maximum power occurs is less than 90^0.

5.6 POWER ANGLE RELATIONSHIP IN A SMIB SYSTEM:

Without loss of generality, many important conclusions on stability can be arrived at by considering the simple case of a Single Machine Infinite Bus (SMIB), where a generator supplies power to an infinite bus. The concept of an infinite bus arises from the fact that if we connect a generator to a much larger power system, it is reasonable to assume that the voltage and frequency of the larger system will not be affected by control of the generator parameters. Hence, the external system can be approximated by an infinite bus,

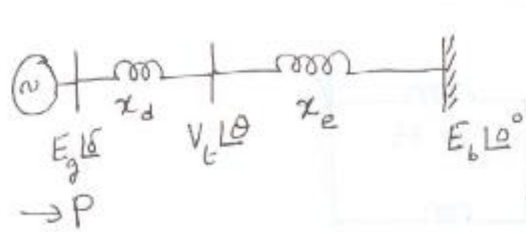


Fig. 5.5 SMIB System

In Fig. 5.5, the infinite bus voltage is taken as reference and δ is the angle between E_g and E_b. The generator is assumed to be connected to the infinite bus through a lossless line of reactance x_e. The power transferred (using classical model) is given by

$$P = \frac{E_g E_b}{x_d + x_e} \sin\delta \tag{5.43}$$

and using salient pole model,

$$P = \frac{E_g E_b}{x_d + x_e} \sin\delta + \frac{E_b^2 \left(x_d - x_q\right)}{2\left(x_d + x_e\right)\left(x_q + x_e\right)} \sin 2\delta \tag{5.44}$$

An important measure of performance is the *steady state stability limit*, which is defined as *the maximum power that can be transmitted in steady state without loss of synchronism*, to the receiving end. If transient analysis is required, respective transient quantities namely $E_g^{'}$, $x_d^{'}$ and $x_q^{'}$ are used in (5.43) and (5.44) to calculate the power output.

5.7 TRANSIENT STABILITY

Transient stability is the ability of the system to remain stable under large disturbances like short circuits, line outages, generation or load loss etc. The evaluation of the transient stability is required offline for planning, design etc. and online for load management, emergency control and security assessment. Transient stability analysis deals with actual solution of the nonlinear differential equations describing the dynamics of the machines and their controls and interfacing it with the algebraic equations describing the interconnections through the transmission network.

Since the disturbance is large, linearized analysis of the swing equation (which describes the rotor dynamics) is not possible. Further, the fault may cause structural changes in the network, because of which the power angle curve prior to fault, during the fault and post fault may be different (See example 9.8). Due to these reasons, a general stability criteria for transient stability cannot be established, as was done in the case of steady state stability (namely $P_S > 0$). Stability can be established, for a given fault, by actual solution of the swing equation. The time taken for the fault to be cleared (by the circuit breakers) is called the *clearing time*. If the fault is cleared fast enough, the probability of the system remaining stable after the clearance is more. If the fault persists for a longer time, likelihood of instability is increased.

Critical clearing time is the maximum time available for clearing the fault, before the system loses stability. Modern circuit breakers are equipped with auto reclosure facility, wherein the breaker automatically recloses after two sequential openings. If the fault still persists, the breakers open permanently. Since most faults are transient, the first reclosure is in general successful. Hence, transient stability has been greatly enhanced by auto closure breakers.

Some common assumptions made during transient stability studies are as follows:

1. Transmission line and synchronous machine resistances are neglected. Since resistance introduces a damping term in the swing equation, this gives pessimistic results.
2. Effect of damper windings is neglected which again gives pessimistic results.
3. Variations in rotor speed are neglected.
4. Mechanical input to the generator is assumed constant. The governor control loop is neglected. This also leads to pessimistic results.
5. The generator is modeled as a constant voltage source behind a transient reactance, neglecting the voltage regulator action.
6. Loads are modeled as constant admittances and absorbed into the bus admittance matrix.

The above assumptions, vastly simplify the equations. A digital computer program for transient stability analysis can easily include more detailed generator models and effect of controls, the discussion of which is beyond the scope of present treatment. Studies on the transient stability of an SMIB system, can shed light on some important aspects of stability of larger systems. One of the important methods for studying the transient stability of an SMIB system is the application of equal-area criterion.

5. 8 EQUAL- AREA CRITERION

Transient stability assessment of an SMIB system is possible without resorting to actual solution of the swing equation, by a method known as equal–area criterion. In a SMIB system, if the system is unstable after a fault is cleared, $\delta(t)$ increases indefinitely with time, till the machine loses synchronism. In contrast, in a stable system, $\delta(t)$ reaches a maximum and then starts reducing as shown in Fig.5.6.

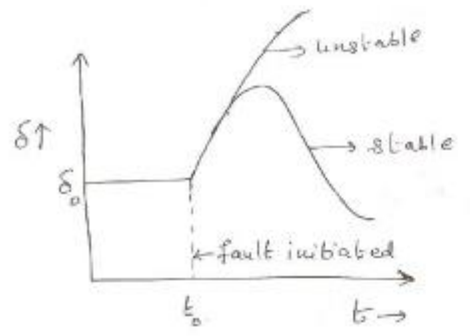


Fig.5.6 Swing Curve (δ Vs t) for stable and unstable system

Mathematically stated,

$$\frac{d\delta(t)}{dt} = 0$$

some time after the fault is cleared in a stable system and $\frac{d\delta}{dt} > 0$, for a long time after the fault is cleared in an unstable system.

Consider the swing equation (21)

$$M\frac{d^2\delta}{dt^2} = P_m - P_e = P_a$$

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

Multiplying both sides by $2\frac{d\delta}{dt}$, we get

$$2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2} = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

This may be written as

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = 2\frac{d\delta}{dt}\frac{P_a}{M}$$

Integrating both sides we get

$$\left(\frac{d\delta}{dt}\right)^2 = \frac{2}{M}\int_{\delta_o}^{\delta} P_a\, d\delta$$

$$\text{or } \frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta_o}^{\delta} P_a\, d\delta} \tag{5.45}$$

For stability $\frac{d\delta}{dt} = 0$, some time after fault is cleared. This means

$$\int_{\delta_o}^{\delta} P_a\, d\delta = 0 \tag{5.46}$$

The integral gives the area under the $P_a - \delta$ curve. The condition for stability can be, thus stated as follows: A SMIB system is stable if the area under the $P_a - \delta$ curve, becomes zero at some value of δ. This means that the accelerating (positive) area under $P_a - \delta$ curve, must equal the decelerating (negative) area under $P_a - \delta$ curve. Application of equal area criterion for several disturbances is discussed next.

5.9 SUDDEN CHANGE IN MECHANICAL INPUT

Consider the SMIB system shown in Fig. 5.7.

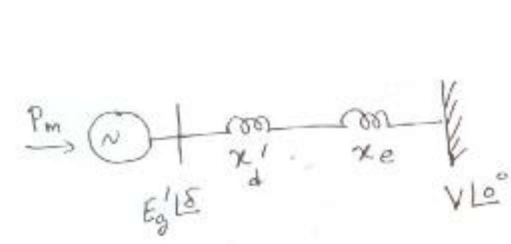


Fig.5.7 SMIB System

The electrical power transferred is given by

$$P_e = P_{max} \sin\delta$$

$$P_{max} = \frac{E_g' V}{x_d' + x_e}$$

Under steady state $P_m = P_e$. Let the machine be initially operating at a steady state angle δ_o, at synchronous speed ω_s, with a mechanical input P_{mo}, as shown in Fig.5.8 (point *a*).

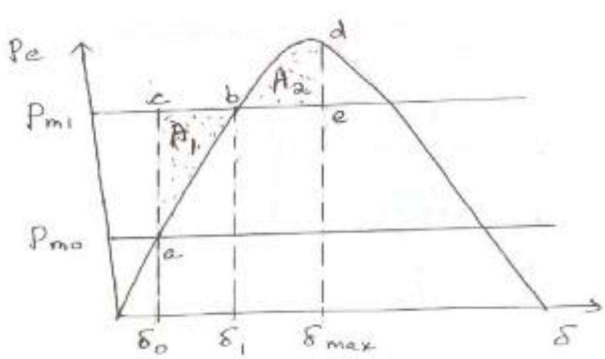


Fig.5.8 Equal area criterion–sudden change in mechanical input

If there is a sudden step increase in input power to P_{m1} the accelerating power is positive (since $P_{m1} > P_{mo}$) and power angle δ increases. With increase in δ, the electrical power P_e increases, the accelerating power decreases, till at $\delta = \delta_1$, the electrical power matches the new input P_{m1}. The area A_1, during acceleration is given by

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e) d\delta$$

$$= P_{m1}(\delta_1 - \delta_0) - P_{max}(\cos\delta_0 - \cos\delta_1) \qquad (5.47)$$

At b, even though the accelerating power is zero, the rotor is running above synchronous speed. Hence, δ and P_e increase beyond b, wherein $P_e < P_{m1}$ and the rotor is subjected to deceleration. The rotor decelerates and speed starts dropping, till at point d, the machine reaches synchronous speed and $\delta = \delta_{max}$. The area A_2, during deceleration is given by

$$A_2 = \int_{\delta_1}^{\delta_{max}} (P_e - P_{m1}) d\delta = P_{max}(\cos\delta_1 - \cos\delta_{max}) - P_{m1}(\delta_{max} - \delta_1) \qquad (5.48)$$

By equal area criterion $A_1 = A_2$. The rotor would then oscillate between δ_0 and δ_{max} at its natural frequency. However, damping forces will reduce subsequent swings and the machine finally settles down to the new steady state value δ_1 (at point b). Stability can be maintained only if area A_2 at least equal to A_1, can be located above P_{m1}. The limiting case is shown in Fig.5.9, where A_2 is just equal to A_1.

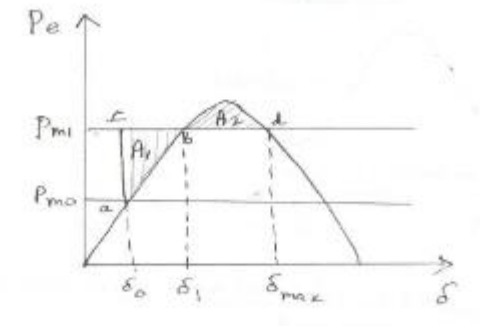


Fig.5.9 Maximum increase in mechanical power

Here δ_{max} is at the intersection of P_e and P_{m1}. If the machine does not reach synchronous speed at *d*, then beyond *d*, P_e decreases with increase in δ, causing δ to increase indefinitely. Applying equal area criterion to Fig.5.9 we get

$A_1 = A_2$.

From (5.47) and (5.48) we get

$$P_{m1}(\delta_{max} - \delta_0) = P_{max}(\cos\delta_0 - \cos\delta_{max})$$

Substituting $P_{m1} = P_{max}\sin\delta_{max}$, we get

$$(\delta_{max} - \delta_0)\sin\delta_{max} + \cos\delta_{max} = \cos\delta_0 \quad (5.49)$$

Equation (5.49) is a non-linear equation in δ_{max} and can be solved by trial and error or by using any numerical method for solution of non-linear algebraic equation (like Newton-Raphson, bisection etc). From solution of δ_{max}, P_{m1} can be calculated. $P_{m1} - P_{mo}$ will give the maximum possible increase in mechanical input before the machine looses stability.

5.10 NUMERICAL EXAMPLES

Example 1: A 50Hz, 4 pole turbo alternator rated 150 MVA, 11 kV has an inertia constant of 9 MJ / MVA. Find the (a) stored energy at synchronous speed (b) the rotor acceleration if the input mechanical power is raised to 100 MW when the electrical load is 75 MW, (c) the speed at the end of 10 cycles if acceleration is assumed constant at the initial value.

Solution:

(a) Stored energy = GH = 150 × 9 = 1350 MJ

(b) $P_a = P_m - P_e = 100 - 75 = 25$ MW

$$M = \frac{GH}{180f} = \frac{1350}{180 \times 50} = 0.15 \ \text{MJ} - \text{s}/^{\circ}\text{e}$$

$$0.15 \frac{d^2\delta}{dt^2} = 25$$

$$\text{Acceleration } \alpha = \frac{d^2\delta}{dt^2} = \frac{25}{0.15} = 166.6 \ ^{\circ}\text{e/s}^2$$

$$= 166.6 \times \frac{2}{P} \ ^{\circ}\text{m/s}^2$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \ rps/\text{s}$$

$$= 166.6 \times \frac{2}{P} \times \frac{1}{360} \times 60 \text{ rpm/s}$$

$$= 13.88 \text{ rpm/s}$$

* Note °e = electrical degree; °m = mechanical degree; P=number of poles.

(c) $10 \text{ cycles} = \frac{10}{50} = 0.2\,\text{s}$

$$N_S = \text{Synchronous speed} = \frac{120 \times 50}{4} = 1500 \text{ rpm}$$

Rotor speed at end of 10 cycles = $N_S + \alpha \times 0.2 = 1500 + 13.88 \times 0.2 = 1502.776$ rpm.

Example 2: Two 50 Hz generating units operate in parallel within the same plant, with the following ratings: Unit 1: 500 MVA, 0.8 pf, 13.2 kV, 3600 rpm: H = 4 MJ/MVA; Unit 2: 1000 MVA, 0.9 pf, 13.8 kV, 1800 rpm: H = 5 MJ/MVA. Calculate the equivalent H constant on a base of 100 MVA.

Solution:

$$H_{1system} = H_{1mach} \times \frac{G_{1mach}}{G_{system}} \quad = 4 \times \frac{500}{100} = 20 \ \text{MJ/MVA}$$

$$H_{2system} = H_{2mach} \times \frac{G_{2mach}}{G_{system}} \quad = 5 \times \frac{1000}{100} = 50 \ \text{MJ/MVA}$$

$H_{eq} = H_1 + H_2 = 20 + 50 = 70$ MJ/MVA

This is the equivalent inertia constant on a base of 100 MVA and can be used when the two machines swing coherently.

Example 3: Obtain the power angle relationship and the generator internal emf for (i) classical model (ii) salient pole model with following data: $x_d = 1.0$ pu : $x_q = 0.6$ pu : $V_t = 1.0$ pu : $I_a = 1.0$ pu at upf

Solution:

(i) **Classical model**: The phasor diagram is shown in Fig P3.

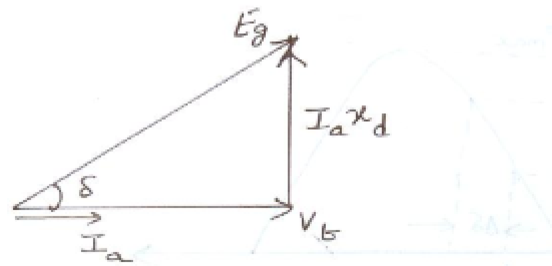


Fig.P3 Example 3, case(i)

$$|E_g| = \sqrt{V_t^2 + (I_a x_d)^2} = \sqrt{(1.0)^2 + (1.0 \times 1.0)^2} = 1.414$$

$$\delta = \tan^{-1}\frac{I_a x_d}{V_t} = \tan^{-1}\frac{1.0}{1.0} = 45°$$

$\therefore$ $E_g = 1.414 \angle 45°$.

If the excitation is held constant so that $|E_g| = 1.414$, then power output

$$P = \frac{1.414 \times 1.0 \sin\delta}{1.0} = 1.414 \sin\delta$$

(ii) **Salient pole:** From Fig (5), we get using (41a) to (41d)

$$E_g = E_q + I_d x_d = V_t \cos\delta + I_d x_d$$

$$= V_t \cos\delta + I_a \sin\delta \, x_d$$

(* $\theta = 0^0$, since we are considering upf)

Substituting given values we get

$$E_g = \cos\delta + \sin\delta.$$

Again from Fig (9.5) we have

$$E_d = V_t \sin\delta - I_q x_q$$

$$\therefore\ V_t \sin\delta - I_q x_q = 0$$

$$V_t \sin\delta - I_a \cos\delta\, x_q = 0$$

Substituting the given values we get

$$0 = \sin\delta - 0.6\cos\delta$$

We thus have two simultaneous equations.

$$E_g = \cos\delta + \sin\delta$$

$$0 = \sin\delta - 0.6\cos\delta$$

Solving we get $\delta = 30.96^\circ$

$$E_g = 1.372 \text{ pu}$$

If the excitation is held constant, then from (42)

$$P = 1.372\sin\delta + 0.333\sin 2\delta$$

Example 4: Determine the steady state stability limit of the system shown in Fig 8, if V_t = 1.0 pu and the reactances are in pu.

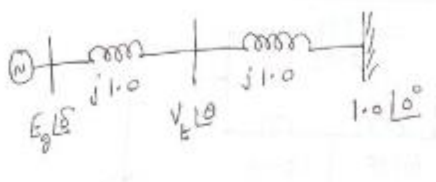


Fig. P4 Example 4

Solution:

$$\text{Current I} = \frac{V_t\angle\theta - 1.0\angle 0^\circ}{j1.0} = \frac{1.0\angle\theta - 1.0\angle 0^\circ}{j1.0}$$

$$E_g\angle\delta = V_t\angle\theta + j1.0(I)$$

$$= 1\angle\theta + \frac{j1.0(1.0\angle\theta - 1.0\angle 0^\circ)}{j1.0}$$

$$= \cos\theta + j\sin\theta + \cos\theta + j\sin\theta - 1.0$$

$$= 2\cos\theta - 1 + j\,2\sin\theta$$

When maximum power is transferred $\delta = 90^\circ$; which means real part of $E = 0$

$$\therefore\ 2\cos\theta - 1 = 0$$

$$\theta = \cos^{-1} 0.5 = 60^\circ$$

$$|E_g| = 2\times\sin 60^\circ = 1.732$$

$$E_g = 1.732\angle 90^\circ \text{ (for maximum power)}$$

$$\text{Steady state stability limit} = \frac{1.732\times 1.0}{1.0+1.0} = 0.866 \text{ pu}$$

Example 5: A 50 Hz synchronous generator having an internal voltage 1.2 pu, H = 5.2 MJ/MVA and a reactance of 0.4 pu is connected to an infinite bus through a double circuit line, each line of reactance 0.35 pu. The generator is delivering 0.8pu power and the infinite bus voltage is 1.0 pu. Determine: maximum power transfer, Steady state operating angle, and Natural frequency of oscillation if damping is neglected.

Solution: The one line diagram is shown in Fig P5.

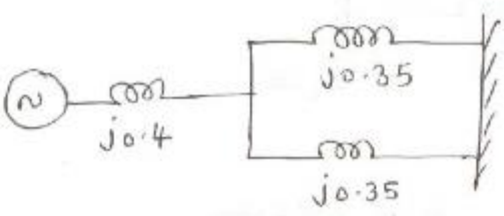


Fig . P5 Example 6

$$\text{(a) } X = 0.4 + \frac{0.35}{2} = 0.575 \text{ pu}$$

$$P_{max} = \frac{E_g\, E_b}{X} = \frac{1.2 \times 1.0}{0.575} = 2.087 \text{ pu}$$

(b) $P_e = P_{max} \sin \delta_o$

$$\therefore \delta_o = \sin^{-1}\frac{P_e}{P_{max}} = \sin^{-1}\left(\frac{0.8}{2.087}\right) = 22.54^\circ.$$

(c) $P_s = P_{max} \cos \delta_0 = 2.087 \cos (22.54^0)$

$= 1.927$ MW (pu)/ elec rad.

$$M\text{ (pu)} = \frac{H}{\Pi f} = \frac{5.2}{\Pi \times 50} = 0.0331 \; s^2 / elec \;\; rad$$

Without damping $s = \pm j\sqrt{\frac{P_s}{M}} = \pm j\sqrt{\frac{1.927}{0.0331}}$

$= \pm j\ 7.63$ rad/sec $=$ 1.21 Hz

Natural frequency of oscillation $\omega_n = 1.21$ Hz.

Example 6: In example .6, if the damping is 0.14 and there is a minor disturbance of $\Delta\,\delta = 0.15$ rad from the initial operating point, determine: (a) ω_n (b) ξ (c) ω_d (d) setting time and (e) expression for δ.

Solution:

(a) $\omega_n = \sqrt{\frac{P_s}{M}} = \sqrt{\frac{1.927}{0.0331}} = 7.63$ rad/sec $= 1.21$ Hz

(b) $\xi = \frac{D}{2}\sqrt{\frac{1}{M\,P_s}} = \frac{0.14}{2}\sqrt{\frac{1}{0.0331 \times 1.927}} = 0.277$

(c) $\omega_d = \omega_n\sqrt{1-\xi^2} = 7.63\sqrt{1-(0.277)^2} = 7.33$ rad/sec $= 1.16$ Hz

(d) Setting time $= 4\tau = 4\frac{1}{\xi\omega_n} = 4 \times \frac{1}{0.277 \times 7.63} = 1.892$ s

(e) $\Delta\,\delta_0 = 0.15 \text{ rad} = 8.59^0$

$\theta = \cos^{-1}\xi = \cos^{-1} 0.277 = 73.9^0$

$$\delta = \delta_o + \frac{\Delta\delta_o}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t} \sin\left(\omega_d t + \theta\right)$$

$$= 22.54^o + \frac{8.59}{\sqrt{1 - 0.277^2}}\, e^{-0.277 \times 7.63t} \sin\left(7.33t + 73.9^o\right)$$

$= 22.54^{0} + 8.94\, e^{-2.11t} \sin(7.33t + 73.9^{0})$

The variation of delta with respect to time is shown below. It can be observed that the angle reaches the steady state value of 22.54° after the initial transient. It should be noted that the magnitudes of the swings decrease in a stable system with damping.

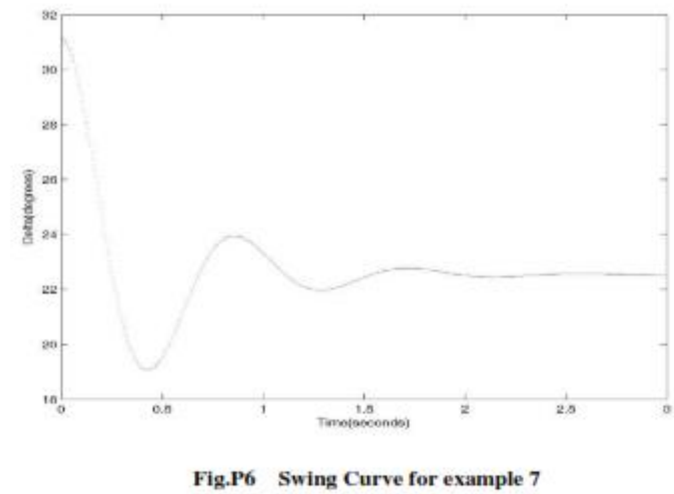


Fig.P6 Swing Curve for example 7

Example 7: In example 6, find the power angle relationship

(i) For the given network

(ii) If a short circuit occurs in the middle of a line

(iii) If fault is cleared by line outage

Assume the generator to be supplying 1.0 pu power initially.

Solution:

(i) From example 6, $P_{max} = 2.087$, $P_e = 2.087 \sin \delta$.

(ii) If a short circuit occurs in the middle of the line, the network equivalent can be draw as shown in Fig. 12a.

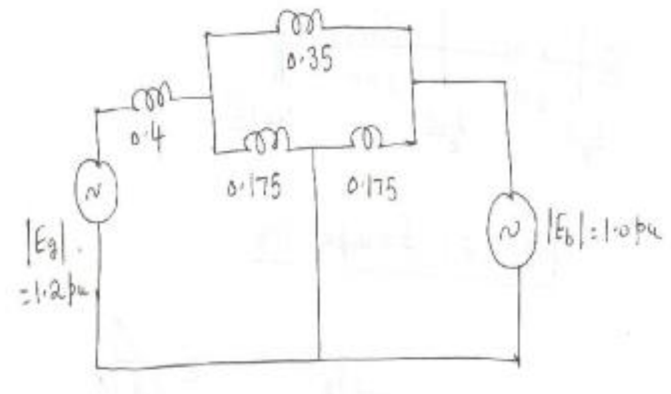
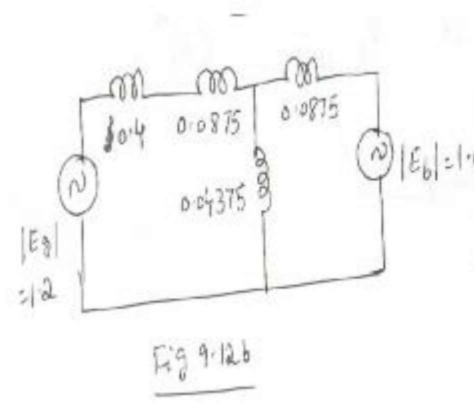


Fig.P7a Short circuit in middle of line

The network is reduced by converting the delta to star and again the resulting star to delta as shown in Fig P7a, P7b and P7c.



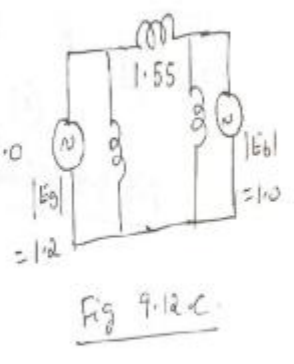


Fig.P7b **Fig.P7c**

The transfer reactance is 1.55 pu. Hence,

$$P_{max} = \frac{1.2 \times 1.0}{1.55} = 0.744 \;;\quad P_e = 0.744 \sin \delta$$

(iii) When there is a line outage

$$X = 0.4 + 0.35 = 0.75$$

$$P_{max} = \frac{1.2 \times 1.0}{0.75} = 1.6$$

$$P_e = 1.6 \sin \delta$$

Example 8: A generator supplies active power of 1.0 pu to an infinite bus, through a lossless line of reactance $x_e = 0.6$ pu. The reactance of the generator and the connecting transformer is 0.3 pu. The transient internal voltage of the generator is 1.12 pu and infinite bus voltage is 1.0 pu. Find the maximum increase in mechanical power that will not cause instability.

Solution:

$$P_{max} = \frac{1.12 \times 1.0}{0.9} = 1.244 \text{ pu}$$

$$P_{mo} = P_{eo} = 1.0 = P_{max} \sin \delta_o = 1.244 \sin \delta_o$$

$$\therefore\ \delta_o = \sin^{-1} \frac{1.0}{1.244} = 53.47^0 = 0.933 \text{ rad.}$$

The above can be solved by N–R method since it is of the form $f(\delta_{max}) = K$. Applying N–R method, at any iteration 'r', we get

$$\Delta\, \delta_{max}^{(r)} = \frac{K - f\left(\delta_{max}^{(r)}\right)}{\dfrac{df}{d\delta_{max}^{(r)}}}$$

$$\frac{df}{d\delta_{max}^{(r)}} = \left(\delta_{max}^{(r)} - \delta_o\right)\cos\delta_{max}^{(r)}$$

(This is the derivative evaluated at a value of $\delta = \delta_{max}^{(r)}$) $\delta_{max}^{(r+1)} = \delta_{max}^{(r)} + \Delta\, \delta_{max}^{(r)}$

Starting from an initial guess of δ_{max} between $\frac{\pi}{2}$ to π, the above equations are solved iteratively till $\Delta\, \delta_{max}^{(r)} \leq \in$. Here $K = \cos \delta_0 = 0.595$. The computations are shown in table P8, starting from an initial guess $\delta_{max}^{(1)} = 1.745$ rad.

Table P8

Interaction r	$\delta_{max}^{(r)}$	$\frac{df}{d\delta_{max}}^{(r)}$	$f(\delta_{max}^{(r)})$	$\Delta\delta_{max}^{(r)}$	$\delta_{max}^{(r+1)}$
1	1.745	-0.1407	0.626	0.22	1.965
2	1.965	-0.396	0.568	-0.068	1.897
3	1.897	-0.309	0.592	-0.0097	1.887
4	1.887	-0.2963	0.596	-0.0033	1.883

Since $\Delta\delta_{max}^{(r)}$ is sufficient by small, we can take

$\delta_{max} = 1.883$ rad $= 107.88^\circ$

$\delta_1 = 180 - \delta_{max} = 72.1^\circ$

$P_{m1} = P_{max} \sin \delta_{max} = 1.183$

Maximum step increase permissible is $P_{m1} - P_{mo} = 1.183 - 1.0 = 0.183$ pu

Example 9: Transform a two machine system to an equivalent SMIB system and show how equal area criterion is applicable to it.

Solution: Consider the two machine system show in Fig.P9.

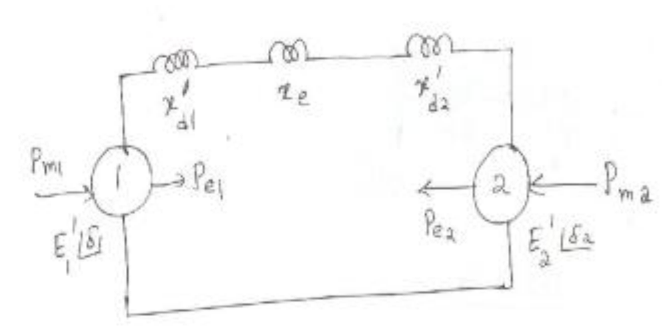


Fig.P9 Two machine system under steady state (neglecting losses)

$P_{m1} = -P_{m2} = P_m$; $P_{e1} = -P_{e2} = P_e$

The swing equations are

$$\frac{d^2\delta_1}{dt^2} = \frac{P_{m1} - P_{e1}}{M_1} = \frac{P_m - P_e}{M_1}$$

$$\frac{d^2\delta_2}{dt^2} = \frac{P_{m2} - P_{e2}}{M_2} = \frac{P_e - P_m}{M_2}$$

Simplifying, we get

$$\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{M_1 + M_2}{M_1 M_2}(P_m - P_e)$$

or $$M_{eq}\frac{d^2\delta}{dt^2} = P_m - P_e$$

where $M_{eq} = \dfrac{M_1 M_2}{M_1 + M_2}$

$$\delta = \delta_1 - \delta_2$$

$$P_e = \frac{E_1' E_2'}{x_{d1}' + x_e + x_{d2}'}\sin\delta$$

This relation is identical to that of an SMIB system in form and can be used to determine the relative swing ($\delta_1 - \delta_2$) between the two machines to assess the stability.

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16. University previous Question papers

Code No: 09A60203 **R09** **Set No. 1**

III B.Tech II Semester Examinations,April/May 2012
COMPUTER METHODS IN POWER SYSTEMS
Electrical And Electronics Engineering

Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
? ? ? ? ?

1. Three generators are rated as follows: Generator 1:100 MVA, 33 kV, reactance 10%, Generator 2:150 MVA, 32 kV, reactance 8% and Generator 3:110 MVA, 30 kV, reactance 12%. Determine the reactance of the generators corresponding to base values of 200 MVA and 35 kV. [15]

2. (a) Define the following terms with suitable example.
    i. Graph
    ii. Tree
    iii. Co-Tree
    iv. Cut-set
    v. Basic Loop.

   (b) Explain the incidence matrices: $\hat{A}$, A, B and C. [7+8]

3. Differentiate between steady state stability and transient stability of a power system. Discuss the factors that affect:
   (a) steady state stability, and
   (b) transient state stability of the system. [15]

4. Develop load flow equations suitable for solution by N-R method using rectangular coordinates when only PQ buses are present. [15]

5. The following is the system data for load flow solution. The line admittances are given in table 1 and active and reactive powers are given in table 2.

Bus Code	P, pu	Q, pu	V, pu	Remarks
1	-	-	1+j0	Slack bus
2	1	0.1	-	PQ bus
3	3.5	0.3	-	PQ bus

Table 2:

Bus Code	Impedance
1-2	-j5
1-3	-j5
2-3	-j10

Table 1:

Find the voltages at the end of first iteration by using G-S method. [15]

6. A power plant has two generators of 10 MVA. 15% reactance each and two 5 MVA generators of 10% reactance paralleled at a common bus bar from which load is taken through a number of 4 MVA step up transformers each having a reactance of 5%. Determine the short circuit capacity of the breakers on the:

   (a) low voltage, and

   (b) high voltage side of the transformer. [15]

7. For the 3-bus system shown in figure 3 obtain Zbus. [15]

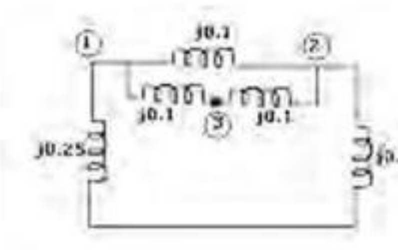

Figure 3:

8. (a) Explain breifly the two forms of instability in power system.

   (b) Does over compensation of a transmission line affects the stability of a power sytem? Justify the answer. [7+8]

? ? ? ? ?

III B.Tech II Semester Examinations,April/May 2012

COMPUTER METHODS IN POWER SYSTEMS

Electrical And Electronics Engineering

Time: 3 hours Max Marks: 75

Answer any FIVE Questions

All Questions carry equal marks

? ? ? ? ?

1. (a) Distinguish between steady state and dynamic stability of a power system network.
   (b) What is meant by power angle curve and write its significance.
   (c) How can the steady state stability of power system be increased? [5+5+5]
2. (a) For the power system network shown in figure 1, draw
      i. Graph
      ii. Tree
      iii. Co-Tree
      iv. Basic loops
      v. Basic cut-sets.
   (b) Write the network performance equations. [7+8]

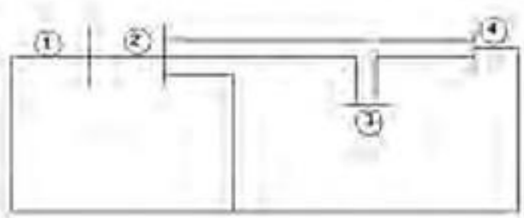

Figure 1:

3. Explain the p.u. system of analyzing power system problems. Discuss the advantages of this method over the absolute method of analysis. [15]
4. A synchronous generator is operating at an infinite bus and supplying 45% of its peak power capacity. As soon as a fault occurs, the reactance between the generator and the line becomes four times its value before the fault. The peak power that can be delivered after the fault is cleared is 70% of the original maximum value. Determine the critical clearing angle. [15]
5. A 65-MVA star-connected 16 kV synchronous generator is connected to 20kV/120 kV, 75 MVA Δ/Y transformer. The sub-transient reactance of the machine is 0.32 p.u. and the reactance of transformer is 0.1 p. u. When the machine is unloaded, a 3-phase fault takes place on the HT side of the transformer. Determine:

(a) the sub transient symmetrical fault current on both sides of the transformer,

(b) the maximum possible value of the d.c. current. Assume 1 p.u. generator voltage. [15]

6. (a) Write the algorithm for FDLF method.

   (b) Compare G-S method and N-R methods. [8+7]

7. Consider the 3-bus system shown in figure 2. The PU line reactances are indicated on the fig. The line resistances are negligible. The magnitudes of all the three bus voltages are specified to be $|V_1| - 1.00$ pu, $|V_2| - 1.04$ pu, $|V_3| - 0.96$ pu. The bus powers are specified in below table .

Bus	Real demand	Reactive demand	Real generation	Reactive generation
1	Pd1−1.0	Qd1−0.6	Pg1−0.7	Qg1 (unspecified)
2	Pd2−0	Qd2−0	Pg2−1.4	Qg2 (unspecified)
3	Pd3−1.0	Qd3−1.0	Pg4−0	Qg3 (unspecified)

Carry out the complete approximate load flow solution. Take bus-1 as slack bus. [15]

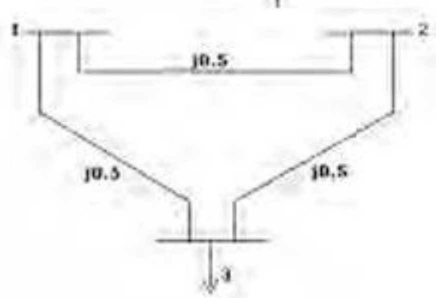

Figure 2:

8. The bus impedance matrix for a 3-bus system is

$$Z_{bus} - \begin{bmatrix} j\,0.3 & j\,0.2 & j\,0.275 \\ j\,0.2 & j\,0.4 & j\,0.25 \\ j\,0.275 & j\,0.25 & j\,0.418 \end{bmatrix}$$

There is a line outage and the line from 1 to 2 is removed. Using the method of building algorithm, determine the new bus impedance matrix. [15]

? ? ? ? ?

III B.Tech II Semester Examinations,April/May 2012
COMPUTER METHODS IN POWER SYSTEMS
Electrical And Electronics Engineering

Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
? ? ? ? ?

1. Two generators rated at 10 MVA 13.2 kV and 15 MVA 13.2kV are connected in parallel to a bus bar. They feed supply to two motors of inputs 8 MVA and 12 MVA respectively. The operating voltage of motors is 12.5 kV. Assuming base quantities as 50 MVA and 13.8 kV draw the reactance diagram. The percent reactance for generators is 15% and that for motors is 20%. [15]

2. (a) Derive the static load flow equations of a n-bus system.
   (b) Explain the advantages and disadvantages of G-S method. [8+7]

3. A motor is receiving 25% of the power that it is capable of receiving from an infinite bus. If the load on the motor is doubled, calculate the maximum value of load angle during the swinging of the rotor around its new equilibrium position. [15]

4. (a) Compare G-S method and N-R methods.
   (b) Write the algorithm for N-R method using rectangular coordinates when PV buses are absent. [7+8]

5. Derive the formulae for Z_{bus} using building algorithm for the addition of link with mutual coupling to other elements. [15]

6. A 50 Hz synchronous generator is connected an infinite bus through a line. The p.u. reactances of generator and the line are j0.2 p.u. and j0.4 p.u. respectively. The generator no load voltage is 1.1 p.u. and that of infinite bus is 1.0 p.u. The inertia constant of the generator is 4 MW-sec/MVA. Determine the frequency of natural oscillations if the generator is loaded to 80% of its maximum power transfer capacity and small perturbation in power is given. [15]

7. A 3 phase, 30 MVA, 6.6kV alternator having 10% reactance is connected through a 30 MVA, 6600/33,000 v delta-star connected transformer of 5% reactance to a 33 kV transmission line having a negligible resistance and a reactance of 4 ohms. At the receiving end of the line there is a 30 MVA, 33,000/6600 volt delta-star connected transformer of 5% reactance stepping down the voltage to 6.6 kV. Both the transformers have their neutral solidly grounded. Draw the one-line diagram and the positive, negative and zero sequence networks of this system and determine the fault currents for single line grounded fault at the receiving station L.V. bus bars.For generator assume -ve sequence reactance as 70% that of + ve sequence. [15]

8. (a) For the 3-bus system shown in figure 4, let a new bus (bus no.4) be added with bus no.2 through a transmission line of impedance (0.01+j0.3)p.u. Obtain Y_{bus} for the new system?

   (b) Explain why Y_{bus} is often used in load flow study. [15]

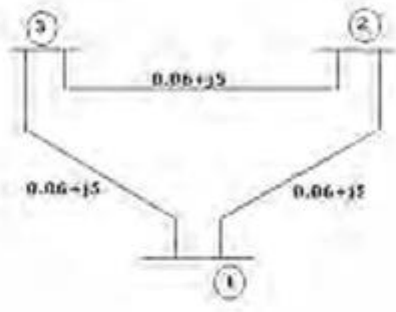

Figure 4:

? ? ? ? ?

Code No: 09A60203 **R09** **Set No. 4**

III B.Tech II Semester Examinations,April/May 2012
COMPUTER METHODS IN POWER SYSTEMS
Electrical And Electronics Engineering

Time: 3 hours Max Marks: 75

Answer any FIVE Questions
All Questions carry equal marks
? ? ? ? ?

1. (a) Wat is the load flow study and explain the need for load flow solution.
   (b) What are the assumptions in SLFE(static load flow equations) and derive the approximate load flow equations. [6+9]
2. Four bus bar sections have each a generator of 40 MVA 10% reactance and a bus bar reactor of 8% reactance. Determine the maximum MVA fed into a fault on any bus bar section and also the maximum MVA if the number of similar bus bars in sections is very large. [15]
3. (a) What are the assumptions in FDLF method?
   (b) Compare the different methods of load flow techniques. [3+12]
4. (a) Derive the formula for power transfer through a transmission line.
   (b) A 4-pole ,50 Hz, 22 kV turbo alternator has a rating of 100 MVA,p.f 0.8 lag.The moment of inertia of rotor is 9000 kg-m^2. Determine M and H. [7+8]
5. A 50 Hz, three-phase synchronous generator delivers 1.00 p.u. power to an infinite busbar through a network in which resistance is negligible. A fault occurs which reduces the maximum power transferable to 0.40 p.u. whereas, before the fault, this power was 1.8 p.u. and, after the clearance of the fault 1.30 p.u. By the use of equal area criterion, determine the critical angle. [15]
6. (a) Prove that when there is no mutual coupling, the diagonal and off-diagonal elements of Y_{Bus} can be computed from $Y_{ii} = \Sigma y_{ij}$ and $Y_{ij} = -y_{ij}$.
   (b) Define the terms graph, tree, co-tree, tree branches, and links.
   Write the relation between branches, links & no. of nodes. [7+8]
7. A three-phase transmission line operating at 33 kV and having a resistance and reactance of 5 ohms and 20 ohms respectively is connected to the generating station bus bar through a 5,000 kVA step-up transformer which has a reactance of 6 per cent, which is connected to the bus bar being supplied by two alternators, one 10,000 kVA having 10% reactance, and another 5,000 kVA having 7.5% reactance. Calculate the kVA at a short-circuit fault between phases occurring
   (a) at the high voltage terminals of the transformers
   (b) at load end of transmission line. [15]
8. Explain the algorithm for the addition and removal of lines in power system. [15]

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17. Question Bank

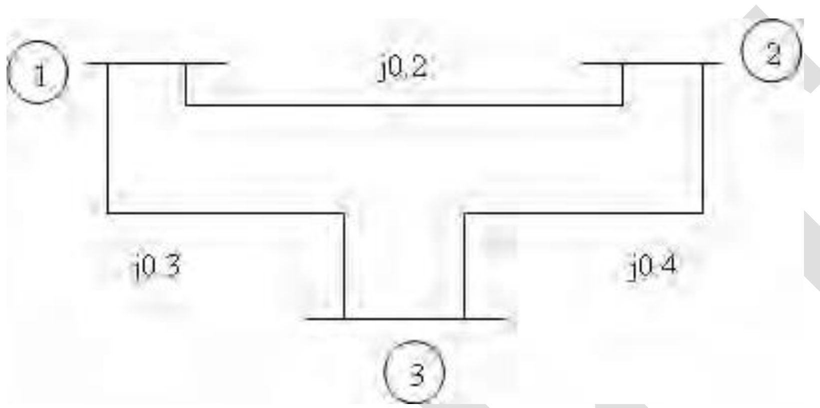
POWER SYSTEM ANALYSIS

Unit – I

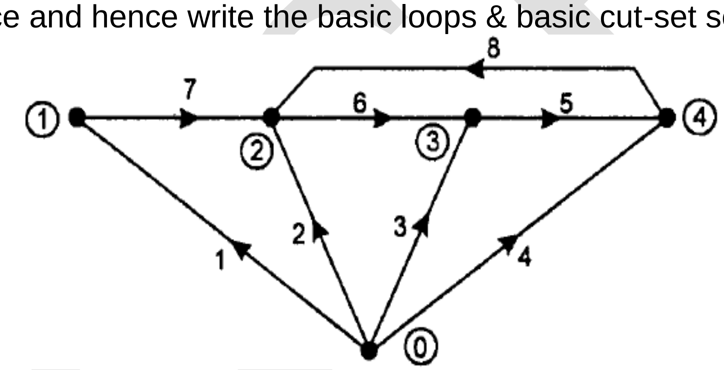
1. Define the following terms with suitable examples

i. Tree ii. Branches iii. Links iv. Co-Tree v. Basic loop vi. Path

2. Form Y_{BUS} for the given power system shown in figure with reactance value in p.u.? Select arbitrary directions.



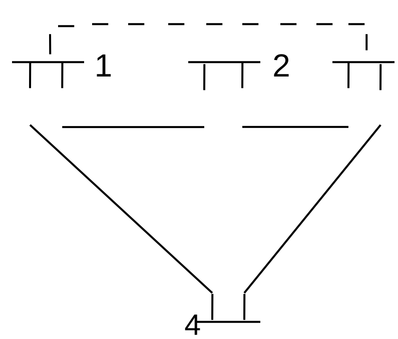
3. For the figure below, draw the tree and the corresponding co-tree. Choose a tree of your
choice and hence write the basic loops & basic cut-set schedule.



4. For a 4-bus system shown the shunt admittances at the buses are neglected and line impedances
are as under:

Line (bus to bus)	1-2	2-3	3-4	1-4
R(pu)	0.025	0.02	0.05	0.04
X(pu)	0.10	0.08	0.20	0.16

3



(a) Assume that the line shown dotted (from bus 1 to bus 3) Is not present.formulate Y_{bus}

(b) Which element of Y_{bus} are affected when the line from bus1 to bus 3 is added. If the pu impedance of this line is 0.1+ j0.4, find the new Y_{bus}

5. Explain the relationship between
    i. The basic loops and links
    ii. Basic cut-sets and the number of branches

Unit – II

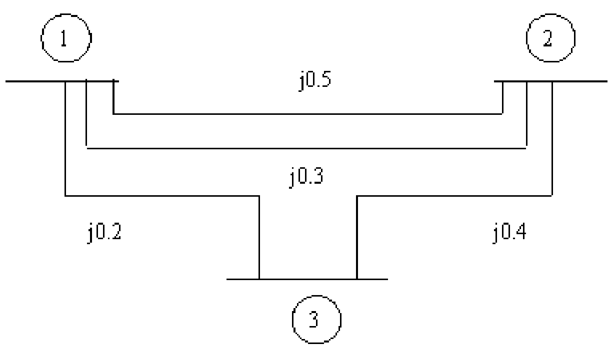
1. Derive the bus admittance matrix by singular transformation method.
2. Represent the power system primitive network component in
    i. Impedance form   ii. Admittance form
3. Build Z_{bus} for the 3-bus system connection given as:

element	**bus code**	**impedance**
1	1-2	j0.1
2	1-2	j0.25
3	1-3	j0.1
4	2-3	j0.1

4. The parameters of a 4-bus system are as under: Find Z_{bus}

Bus code	**Line impedance (pu)**	**Charging admittance $Y_{pq}/2$(pu)**
1-2	0.2+j0.8	j0.02
2-3	0.3+j0.9	j0.03
2-4	0.25+j1	j0.04
3-4	0.2+j0.8	j0.02
1-3	0.1+j0.4	j0.01

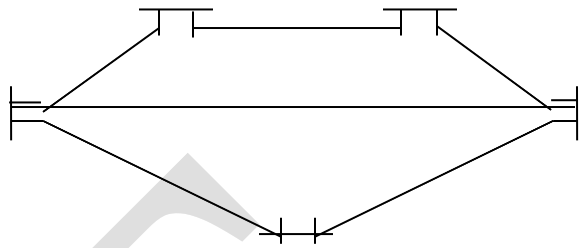
5. For the network as shown in figure. Obtain Z_{bus} take Bus-3 as reference Bus.



Unit – III

1. What are the advantages & disadvantages of
    i.G-S method   ii. N-R method
2. Describe load flow solution with P.V buses using G-S method.
3. Derive the basic equations for load flow studies and also write the assumptions and approximations to get the simple equations.
4. With a flow chart, explain G-S method for load flow studies

5. Below fig shows a five bus system. Each line has an Z=0.05+j0.15pu. the line shunt admittances
may be neglected.The bus power and voltage specifications are given below

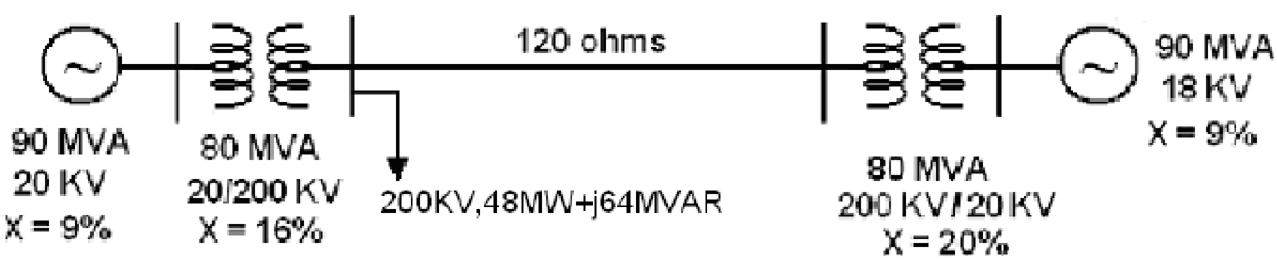


Unit – IV

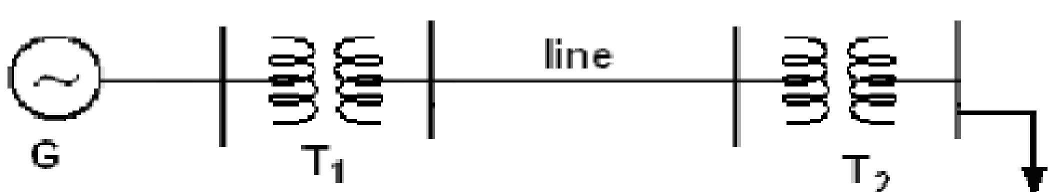
1. Describe the Newton-Raphson method for the solution of power flow equations in power systems deriving necessary equations using rectangular co-ordinates
2. Derive the expression for diagonal and off-diagonal elements of Jacobin matrix of N-R (Polar form) method.
3. With a flow chart, explain Fast decoupled load flow method
4. With a flow chart, explain NR method in polar co-ordinates for load flow studies
5. Discuss the Comparison of different methods.

Unit – 5

1. Prove that $Z_{pu(new)} = Z_{pu(old)} \times \frac{(MVA)_{Base(new)}}{(MVA)_{Base(old)}} \times \frac{(KV_{LL})^2_{Base\ old}}{(KV_{LL})^2_{Base\ old}}$
2. Explain the importance of Per-unit system
3. How are reactors classified? Explain the merits and demerits of different types of system protection using reactors.
4. Draw the pu impedance diagram for the system shown in figure below. Choose Base
MVA as 100 MVA and Base KV as 20 KV.

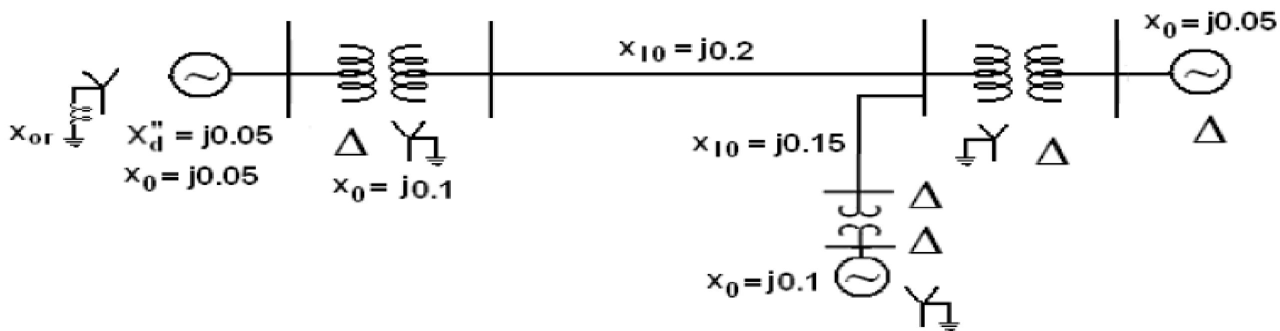


5. Obtain pu impedance diagram of the power system of figure below. Choose base quantities as 15 MVA and 33 KV.
Generator: 30 MVA, 10.5 KV, X'' = 1.6 ohms.
Transformers T_1 & T_2: 15 MVA, 33/11 KV, X = 15 ohms referred to HV
Transmission line: 20 ohms / phase
Load: 40 MW, 6.6 KV, 0.85 lagging p.f.



Unit – 6

1. What are symmetrical components? Explain.
2. Derive an expression for power in a 3-phase circuit in terms of symmetrical components
3. Draw zero sequence network for the system shown in figure below



4.The line currents in a 3 phase supply to an un balanced load are respectively $I_a = 10 + j20$; $I_b = 12 - j10$; $I_c = -3 - j5$ Amp. phase sequence is abc. Determine the sequence components of currents.

4. Derive an expression for the fault current for a line-to-line fault at an unloaded generator.

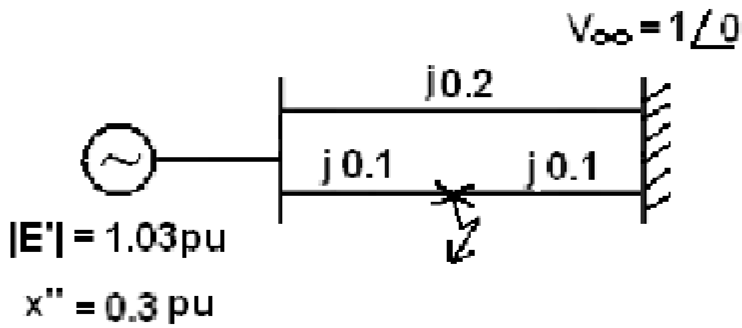
Unit – 7

1. Explain the terms
   a) Steady state stability b) Transient stability c) Dynamic stability
2. Discuss various methods of improving steady state stability & transient stability
3. A 3 phase line is 400 Km long. The line parameters are $r = 0.125$ ohm/Km; $x = 0.4$ ohm/Km and $y = 2.8\text{x}10^{-6}$ mhos/Km. Find steady state stability limit if $|V_S| = |V_R| = 220$KV .
4. A salient pole synchronous generator is connected to an infinite bus via a line. Derive an expression for electrical power output of the generator and draw p-δ curve.
5. What are the factors that affect the transient stability? Explain in detail.

Unit – 8

1. What are the assumptions made in deriving swing equation.
2. Explain point by point method of determine swing curve.
3. Derive and explain the equal area criterion for stability of a power system.

4. Derive the formula for calculating critical clearing angle.
5. For the system shown in figure below, a 3 phase fault occurs at the middle of one of
the transmission lines and is cleared by simultaneous opening of circuit breakers at both ends. If initial power of generator is 0.8 pu, determine the critical clearing angle.



18. Assignment topics

1. Draw the reactance diagram for the power system shown in Fig.1. Neglect resistance and use a base of 100 MVA, 220 kV in 50 Ω line. The ratings of the generator, motor and transformer are given below.

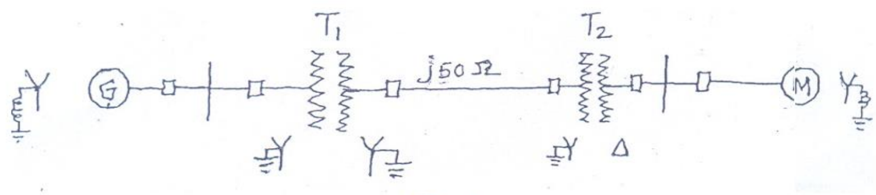


Fig. 1

Generator: 40 MVA, 25 kV, X" = 20%
Synchronous motor : 50 MVA, 11 kV, X" = 30%
Y – Y Transformer : 40 MVA, 33/220 kV, X = 15%
Y - Δ 30 MVA, 11/220 kV, (Δ/Y), X = 15% (16)

2. Draw the structure of an electrical power system and describe the components of the system with typical values (16)
3. Obtain the per unit impedance (reactance) diagram of the power system shown in Fig.3

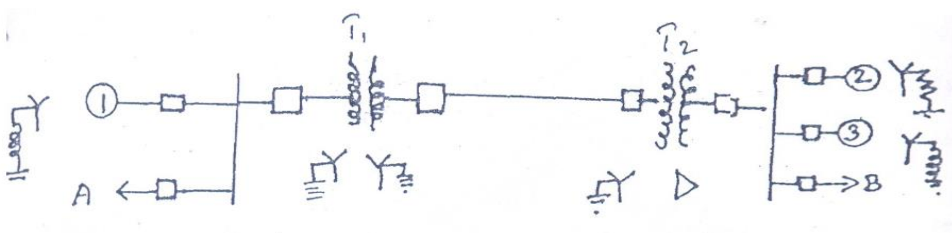


Fig. 3
One-line representation of a simple power system.

Generator No. 1: 30 MVA, 10.5 kV, X" = 1.6 Ohm
Generator No. 2: 15 MVA, 6.6 kV, X" = 1.2 Ohm
Generator No. 3: 25 MVA, 6.6 kV, X" = 0.56 Ohm
Transformer T_1 (3phase) : 15 MVA, 33/11 kV, X = 15.2 Ohm per phase on HT side
Transformer T_2 (3phase) : 15 MVA, 33/6.2 kV, X = 16 Ohm per phase on HT side
Transmission line : 20.5 Ohm/phase
Load A : 15 MW, 11kV, 0.9 p.f. lagging
Load B : 40 MW, 6.6 kV, 0.85 lagging p.f. (16)

4. Explain the modeling of generator, load, transmission line and transformer for power flow, short circuit and stability studies. (16)
5. Choosing a common base of 20 MVA, compute the per unit impedance (reactance) of the components of the power system shown in Fig.5 and draw the positive sequence impedance (reactance) diagram.

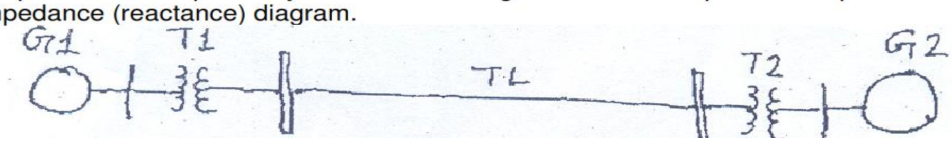


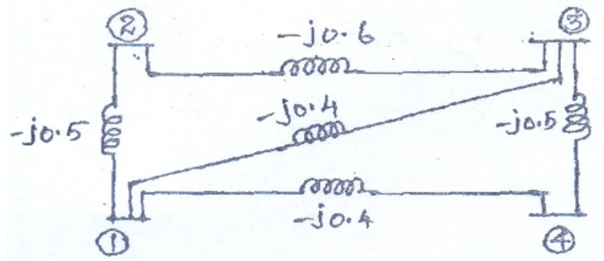
Fig. 5

Gen 1 : 20 MVA, 10.5 kV, X" = 1.4 Ohm
Gen 2 : 10 MVA, 6.6 kV, X" = 1.2 Ohm
Tr 1 : 10 MVA, 33/11 kV, X = 15.2 Ohm per phase on HT side
Tr 2 : 10 MVA, 33/6.2 kV, X = 16.0 Ohm per phase on HT side
Transmission line : 22.5 Ohms per phase (16)

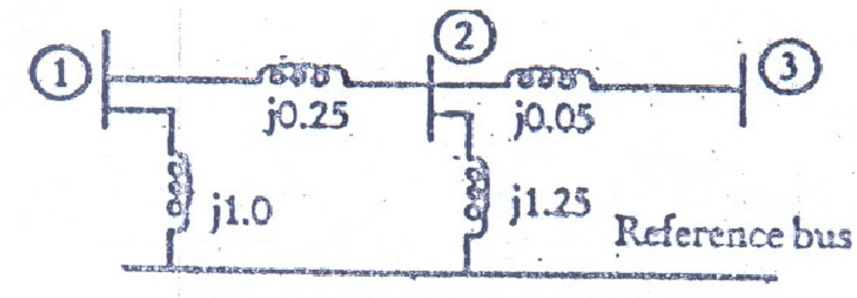
6. (i) What are the step by step procedures to be followed to find the per-unit impedance diagram of a power system? (4)
   (ii) Draw the structure of an electrical power system and describe the components of the system with typical values. (12)
7. Write short notes on:
   (i) Single line diagram (5)
   (ii) Change of base. (5)
   (iii) Reactance of synchronous machines. (6)
8. A 120 MVA, 19.5 kV Generator has a synchronous reactance of 0.15 p.u and it is connected to a transmission line through a Transformer rated 150 MVA, 230/18 kV (star/delta) with X = 0.1 p.u.
   (i) Calculate the p.u reactance by taking generator rating as base values (5)
   (ii) Calculate the p.u reactance by taking transformer rating as base values. (5)
   (iii) Calculate the p.u reactance for a base value of 100 MVA and 220 kV on H.T side of transformer. (6)

---

1. Derive load flow algorithm using Gauss – Seidel method with flow chart and discuss the advantages of the method. (16)
2. Derive load flow algorithm using Newton-Raphson method with flow chart and state the importance of the method. (16)
3. Explain clearly the algorithmic steps for solving load flow equation using Newton – Raphson method (polar form) when the system contains all types of buses. Assume that the generators at the P-V buses have adequate Q Limits. (16)
4. Explain the step by step procedure for the NR method of load flow studies. (16)
5. Find the bus admittance matrix for the given network. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u. (16)



6. Find the bus impedance matrix for the system whose reactance diagram is shown in fig. All the impedances are in p,u. (16)

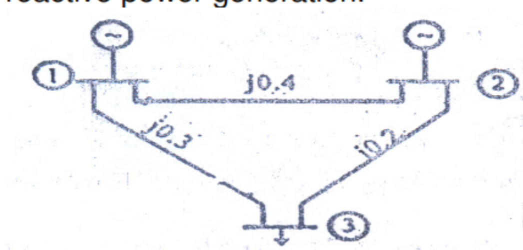


7. (i) Derive the power flow equation in polar form. (8)
   (ii)Write the advantages and disadvantages of Gauss-Seidel method and Newton-Raphson method. (8)
8. The parameters of a 4-bus system are as under:

Bus code	**Line impedance (pu)**	**Charging admittance (pu)**
1-2	0.2 + j 0.8	j 0.02
2-3	0.3 + j 0.9	j 0.03
2-4	0.25 +j 1.0	j 0.04
3-4	0.2 + j 0.8	j 0.02
1-3	0.1 + j0.4	j 0.01

Draw the network and find bus admittance matrix. (16)

9. With a flow chart, explain the NR Iterative method for solving load flow problem. (16)
10. (i) Compare Gauss-Seidel method and Newton-Raphson method of load flow studies (6)
    (ii) Fig.12 shows a three bus power system.
    Bus 1 : Slack bus, V= $1.05/0^0$ p.u.
    Bus 2 : PV bus, V = 1.0 p.u. $P_g = 3$ p.u.
    Bus 3 : PQ bus, $P_l = 4$ p.u., $Q_l = 2$ p.u.
    Carry out one iteration of load flow solution by Gauss Seidel method. Neglect limits on reactive power generation. (10)



1. A generator is connected through a transformer to a synchronous motor the sub transient reactance of generator and motor are 0.15 p.u. and 0.35 p.u. respectively. The leakage reactance of the transformer is 0.1 p.u. All the reactances are calculated on a common base. A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u. The output current of generator is 1 p.u. and 0.8 p.f. leading. Find the sub transient current in p.u. in the fault, generator and motor. Use the terminal voltage of generator as reference vector. (16)

2. Explain the step by step procedure for systematic fault analysis using bus impedance matrix. (16)

3. A 60 MVA, Y connected 11 KV synchronous generator is connected to a 60 MVA, 11/132 KV Δ/Y transformer. The sub transient reactance X''_d of the generator is 0.12 p.u. on a 60 MVA base, while the transformer reactance is 0.1 p.u. on the same base. The generator is unloaded when a symmetrical fault is suddenly placed at point p as shown in Fig. 3 Find the sub transient symmetrical fault current in p.u. amperes and actual amperes on both side of the transformer. Phase to neutral voltage of the generator at no load is 1.0 p.u. (16)

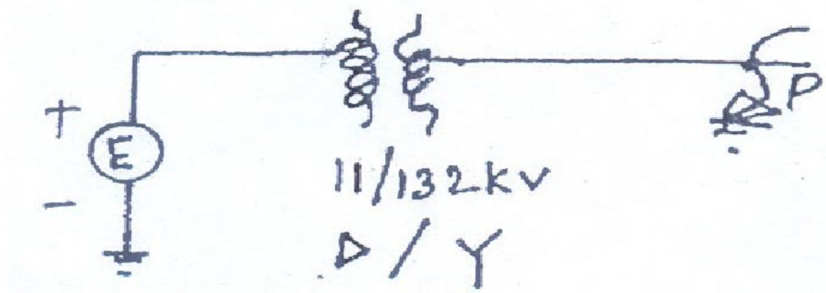
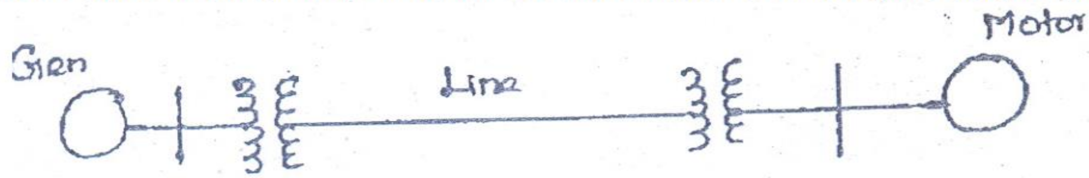


Fig. 3

4. A three –phase transmission line operating at 33 KV and having a resistance and reactance of 5 Ohms and 15 Ohms respectively is connected to the generating station bus-bar through a 5000 KVA step up transformer which has a reactance of 0.05 p.u. Connected to the bus-bars are two alternators, are 10,000 KVA having 0.08 p.u. reactance and another 5000 KVA having 0.06 p.u. reactance. Calculate the KVA at a short circuit fault between phases occurring at the high voltage terminals of the transformers. (16)

5. A synchronous generator and a synchronous motor each rated 25 MVA, 11 KV having 15% sub-transient reactance are connected through transformers and a line as shown in fig. The transformers are rated 25 MVA< 11/66 KV and 66/11 KV with leakage reactance of 10% each. The line has a reactance of 10% on a base of 25 MVA, 66 Kv. The motor is drawing 15 MW at 0.5 power factor leading and a terminal voltage of 10.6 KV. When a symmetrical 3 phase fault occurs at the motor terminals. Find the sub-transient current in the generator, motor and fault.



(16)

6. A three phase power of 700 MW is to be transmitted to a substation located 315 kM from the source of power. For a preliminary line design assume the following parameters:

V_s = 1.0 p.u., V_r + 0.9 p.u. ^ = 5000 km ; z_c = 320 Ω, and Ś = 36.87^0 .

(i) Based on the practical line load ability equation, determine a nominal voltage level for the transmission line. (8)

(ii) For the transmission voltage level obtained in (i) Calculate the theoretical maximum power that can be transferred by the transmission line. (8)

7. A 25,000 KVA, 13.8 kV generator with X''_d = 15% is connected through a transformer to a bus which supplies four identical motors as shown in Fig. 7 The sub transient reactance X''_d of each

motor is 20% on a base of 5000 KVA, 6.9 kV. The three-phase rating of the transformer is 25,000 KVA, 13.8/6.9 kV, with a leakage reactance of 10%. The bus voltage at the motors is 6.9 kV when a three-phase fault occurs at point p. for the fault specified, determine (i) the sub transient current in the fault (ii) the sub transient current in breaker A and (iii) the symmetrical short-circuit interrupting current in the fault and in breaker A. (16)

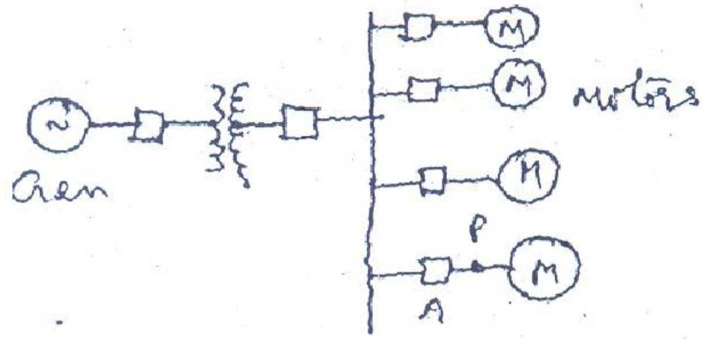


Fig.7 one line diagram

8 Determine Z_{bus} for the network shown below in Fig. 8 where the impedances labeled 1 through 6 are shown in per unit. Preserve all buses. (16)

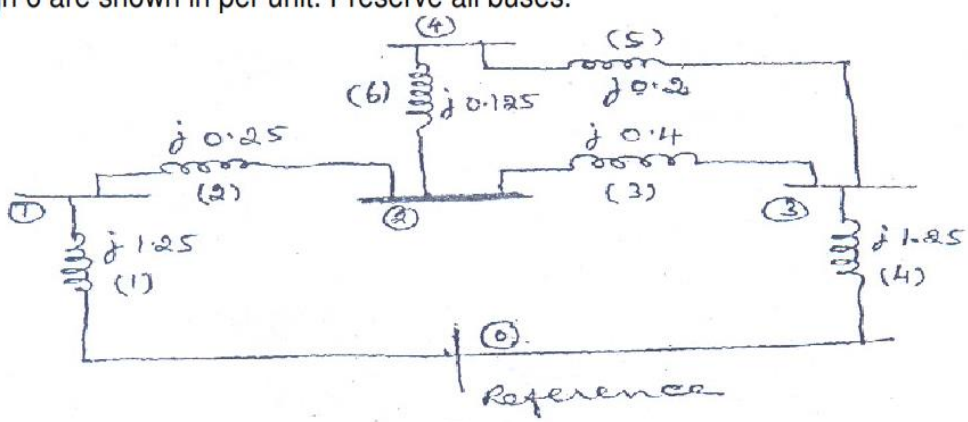
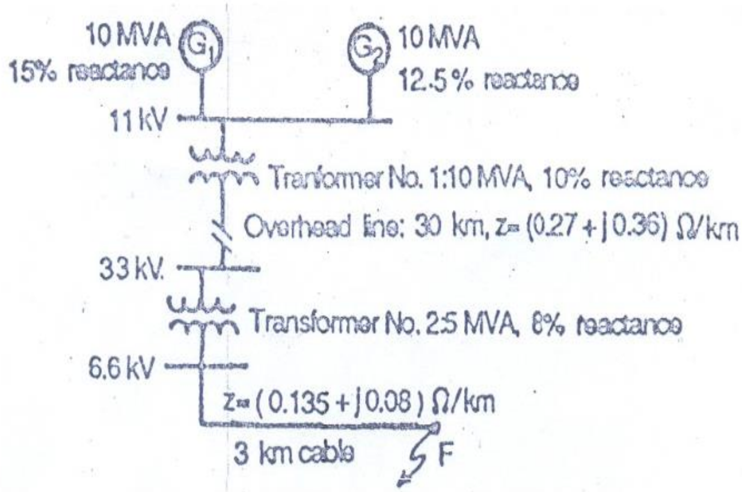


Fig. 8

Fig. 8 Branch impedances are in p.u. and branch numbers are in parentheses.

8. With a help of a detailed flowchart, explain how a symmetrical fault can be analyzed using Z_{bus} ? (16)

8. (i) For the radial network shone below a three phase fault occurs at F. Determine the fault current and the line voltage at 11 kV bus under fault conditions. (6)



(ii) Explain the procedure for making short-circuit studies of a large power system networks using digital computers. (10)

9. Two synchronous machines are connected through three phase transformers to the transmission line shown in Fig.11 the ratings and reactance of the machines and transformers are

Machine 1 and 2 : 100 MVa, 20kV; $X''_d = X_1 = X_2 = 20\%$

$X_0 = 4\%$, $X_n = 5\%$

Transformers T_1 and T_2 : 100 MVA, 20 Δ/345 YkV ; X = 8%.

On a chosen base of 100 MVA, 345 kV in the transmission line circuit the line reactances are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. Draw each of the three sequence networks and find the zero sequence bus impedance matrixes by means of Z_{bus} building algorithm. (16)

The positive sequence impedance of an equipment is the impedance offered by the equipment to the flow of positive sequence currents.

4. Draw the connection of sequence networks for a double line-to-ground fault at the terminals of an unloaded generator.

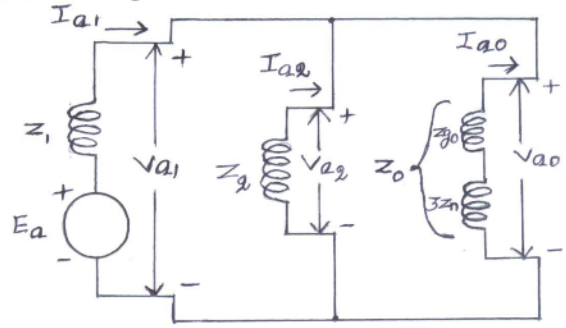


Fig F1: Connection of the sequence Networks of an unloaded generator for A double line to-ground fault on phase b and c

5. Draw the connection of sequence networks for line-to-line fault without fault impedance.

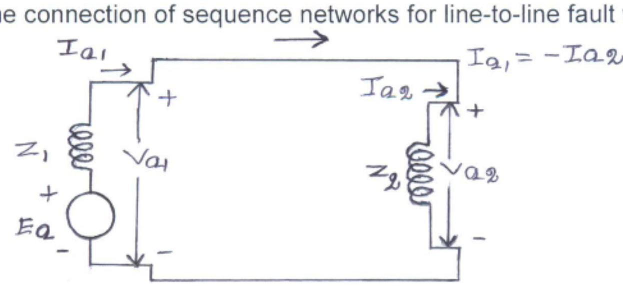


Fig F 2: Connection of the sequence networks For a line-to-line fault in power system.

6. Draw the connection of sequence networks for double line-to-ground fault without fault impedance.

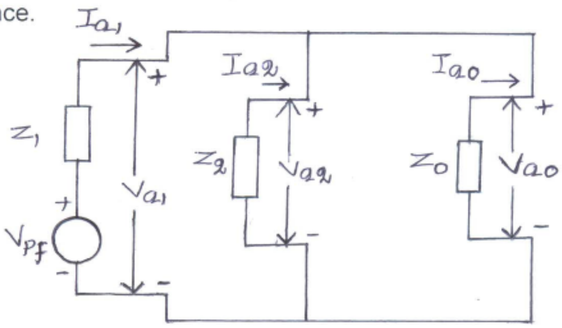


Fig F 3: Connection of the sequence Networks for a double line-to-ground fault

7. Draw the connection of sequence networks for in-to-ground fault through impedance Z_f.

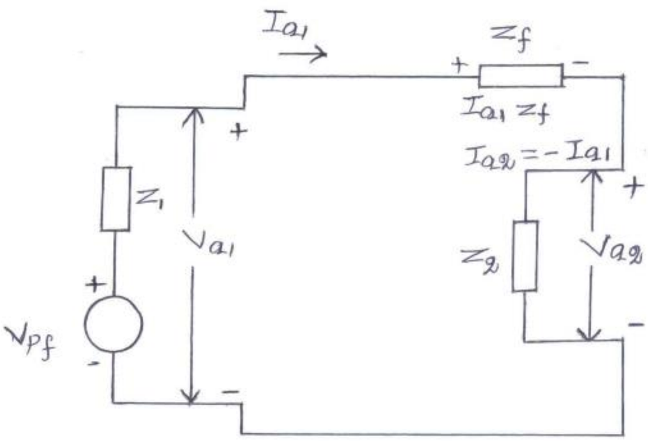


Fig F 4: Connection of sequence networks for Line-to-line fault through impedance Z_f.

8. Draw the connection of sequence networks for double line-to-ground fault through an impedance Z_f.

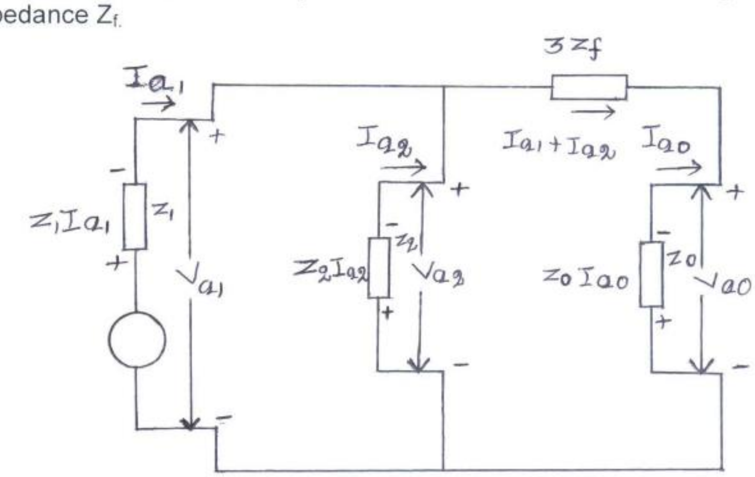


Fig F5: connection of sequence networks for a double Line-to-ground fault through an impedance

2. Determine the fault current and MVA at faulted bus for a line to ground (solid) fault at bus 4 as shown in Fig.2

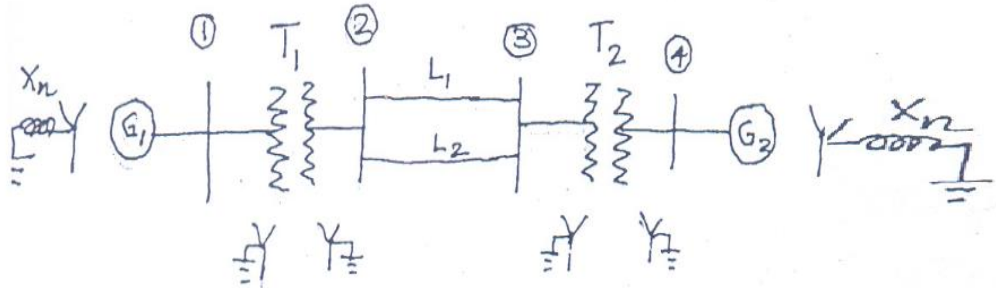


Fig.2

G_1, G_2 : 100 MVA, 11kV, $X^{+} + X^{-} = 15\%$, $X^{0} = 5\%$, $X_n = 6\%$

T_1 T_2 : 100 MVA, 11kV/220 kV, $X_{l\backslash leak} = 9\%$

L_1,L_2 : $X^{+} = X^{-} = 10\%$, $X^{0} = 10\%$ on base of 100 MVA. Consider a fault at phase a'.

(16)

3. A single line to ground fault occurs on bus 4 of the system shown in Fig.3
   (i) Draw the sequence networks and (12)
   (ii) Compute the fault current. (4)

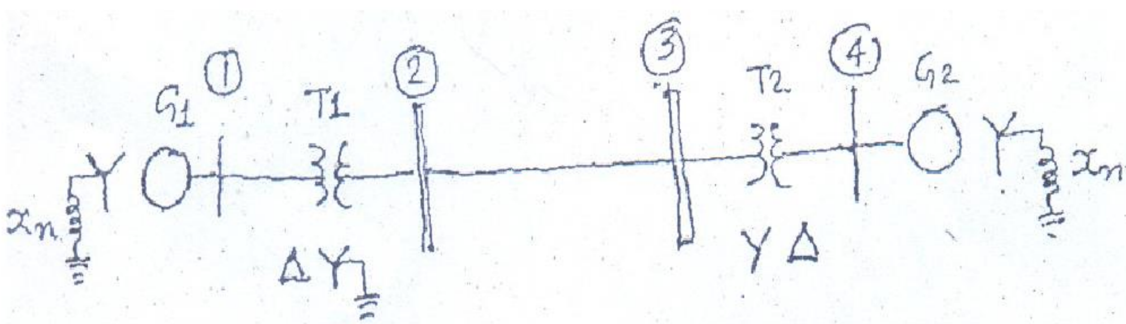


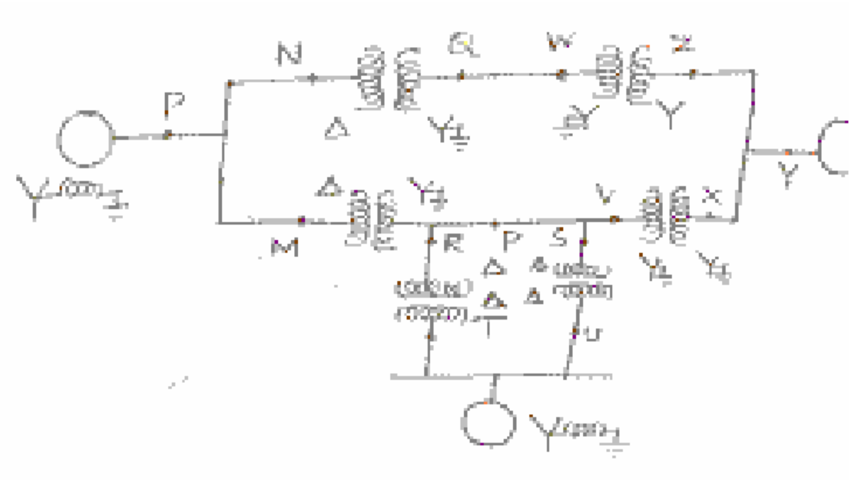
Fig. 3

Gen 1 and 2 : 100 MVA, 20kV; $X' = X''$ 20% ; $X_0 = 4\%$; $X_n = 5\%$.

Transformer 1 and 2 : 100 MVA, 20/345 KV; $X_{leakage} = 8\%$ on 100 MVA

Tr. Line : $X' = X' = 15\%$ $X_0 = 50\%$ on a base of 100 MVA, 20 kV.

4.. Draw the Zero sequence diagram for the system whose one line diagram is shown in fig.



(16)

5. Two synchronous machines are connected through three-phase transformers to the transmission line as given below in Fig. 5. The ratings and reactance of the machines and transformers are
   Machines 1 and 2 : 100 MVA, 20 Kv; $X''_d = X_1 = X_2 = 20\%$
   $X_0 = 4\%$; $X_n = 5\%$.
   Transformers T_1 and T_2 : 100 Mva, 20y/345 YkV ; X= 8%
   Both transformers are solidly grounded on two sides. On a chosen base of 100 MVA, 345 kV in the transmission line circuit the line reactance are $X_1 = X_2 = 15\%$ and $X_0 = 50\%$. The system is operating at nominal voltage without prefault currents when a bolted (Z_f = o) single line-to-ground fault occurs on phase A at bus (3) Using the bus impedance matrix for each of the three sequence networks, determine the sub transient current to ground at the fault. (16)

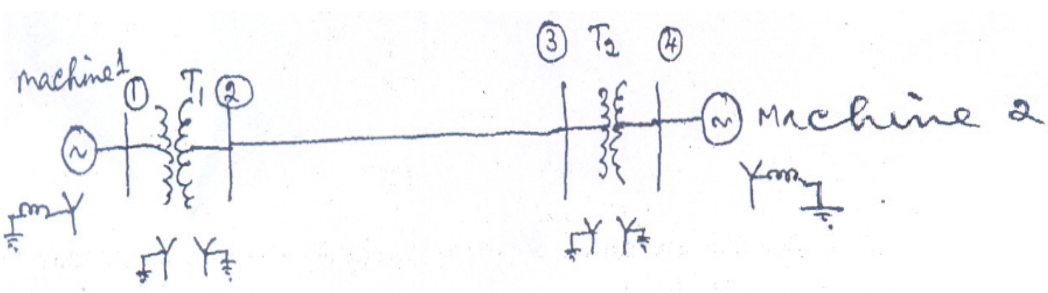


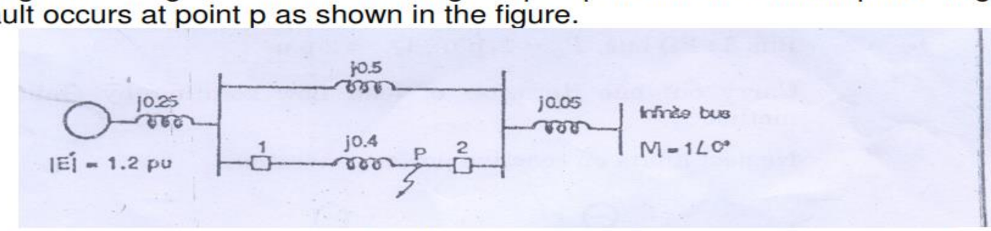
Fig.5

6. Determine the positive, negative and zero sequence networks for the system shown in Fig. 6. Assume zero sequence reactance for the generator and synchronous motors as 0.06 p.u. current limiting reactors of 2.5 Ω are connected in the neutral of the generator and motor No.2 The zero sequence reactance of the transmission line is j 300 Ω.

(10)

1. Derive swing equation used for stability studies in power system. (16)
2. Explain the modified Euler method of analyzing multi machine power system for stability with a neat flow chart. (16)
3. (i) Derive swing equation for a synchronous machine. (8)

   (ii) A 50 H_z generator is delivering 50% of the power that it is capable of delivering through a transmission line to an infinite bus. A fault occurs that increases the reactance between the generator and the infinite bus to 500% of the value before the fault. When the fault is isolated, the maximum power that can be delivered is 75% of the original maximum value. Determine the critical clearing angle for the condition described. (8)
4. Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 p.u. power at the instant preceding the fault. The fault occurs at point p as shown in the figure.



(16)

5 In the system shown in Fig. 5 a three phase static capacitive reactor of reactance 1 p.u. per phase in connected through a switch at motor bus bar. Calculate the limit of steady state power with and without reactor switch closed. Recalculate the power limit with capacitance reactor replaced by an inductive reactor of the same value. (16)

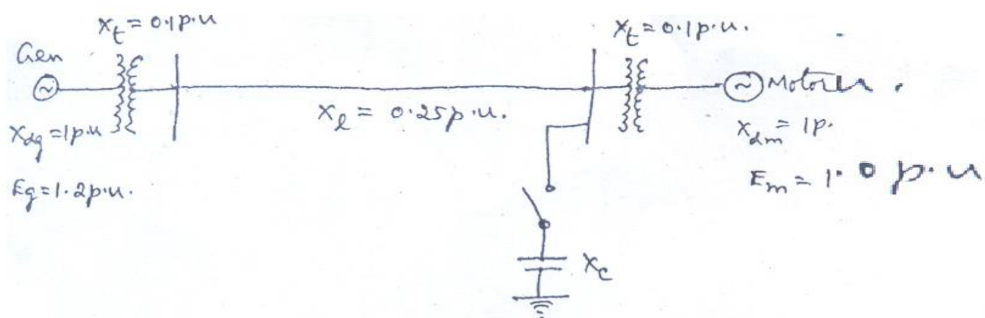


Fig.5

Assume the internal voltage of the generator to be 1.2 pu. and motor to be1.0 p.u.

6. Describe the Runge-Kutta method of solution of swing equation for multi-machine systems. (16)
7. (i) Derive the swing equation of a synchronous machine swinging against an infinite bus. Clearly state the assumption in deducing the swing equation. (10)

   (ii) The generator shown in Fig. 7 is delivering power to infinite bus. Take V_t = 1.1 p.u. Find the maximum power that can be transferred when the system is healthy. (6)

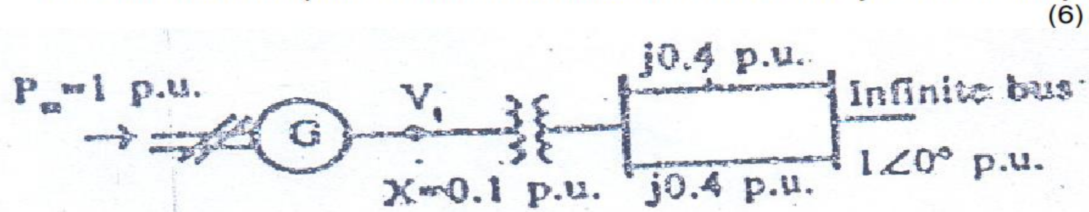
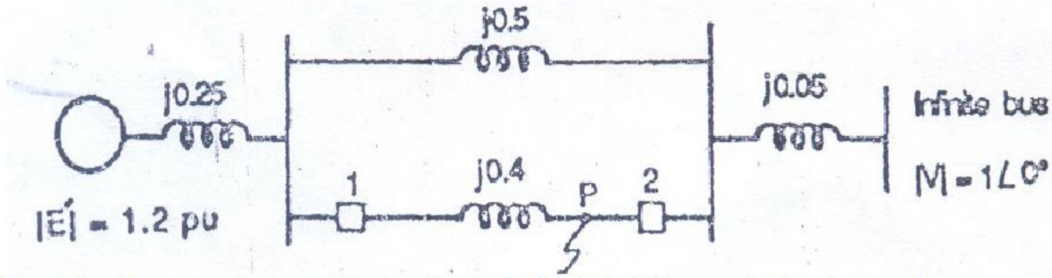


Fig. 7

9. (i) A 2-pole 50 Hz, 11kV turbo alternator has a ratio of 100 MW, power factor 0.85 lagging. The rotor has a moment of inertia of 10,000 kgm^2. Calculate H and M. (6)
(ii)A three phase fault is applied at the point P as shown below. Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated in the diagram. The generator is delivering 1.0 p.u. power at the instant preceding the fault. (10)



10. Describe the equal area criterion for transient stability analysis of a system. (16)

----------- X ---------

19. Quiz Questions .

Objective type questions

1. Under no load conditions the current in a transmission line is due to.
   a) Corona effects
   b) Capacitance of the line
   c) Back flow from earth
   d) None of the above
2. In the short transmission line which of the following is used?
   a) π - Model
   b) T – Model
   c) Both (a) and (b)
   d) None of the above
3. In the short transmission line which of the following is neglected?
   a) I^2 R loss
   b) Shunt admittance
   c) Series impedance
   d) All of the above
4. Which of the following loss in a transformer is zero even at full load?
   a) Eddy current
   b) Hysteresis
   c) Core loss
   d) Friction loss
5. The transmission line conductors are transposed to
   a) Balance the current
   b) Obtain different losses
   c) Obtain same line drops
   d) Balance the voltage

[Ans.: 1(b), 2(a), 3(b), 4(d), 5(c)]

OBJECTIVE TYPE QUESTIONS

1. When a 1-phase supply is across a 1-phase winding, the nature of the magnetic field produced is

- **a)** Constant in magnitude and direction
- **b)** Constant in magnitude and rotating at synchronous speed
- **c)** Pulsating in nature
- **d)** Rotating in nature

2. The damper windings are used in alternators to

- **a)** Reduce eddy current loss
- **b)** Reduce hunting
- **c)** Make rotor dynamically balanced
- **d)** Reduce armature reaction

3. The neutral path impedance Zn is used in the equivalent sequence network models as

- **a)** Zn2
- **b)** Zn
- **c)** 3 Zn
- **d)** An ineffective value

4. An infinite bus-bar should maintain

- **a)** Constant frequency and Constant voltage
- **b)** Infinite frequency and Infinite voltage
- **c)** Constant frequency and Variable voltage
- **d)** Variable frequency and Variable voltage

5. Voltages under extra high voltage are

- **a)** 1KV & above
- **b)** 11KV & above
- **c)** 132 KV & above
- **d)** 330 KV & above

[Ans.: 1(c), 2(b), 3(c), 4(a), 5(d)]

Power System Stability Objective Questions

1. Steady state stability of the power system is improved by
a) Reducing the fault clearance time
b) Using the double circuit line instead of single circuit line
c) single pole switching
d) decreasing generator inertia
Answer
B

2. Equal area criteria gives the information regarding
a) Stability region
b) Absolute Stability
c) Relative Stability
d) Swing Curves
Answer
B

3. Which one of the following is true
a) Steady State Stability limit is greater than Transient Stability limit
b) Steady State Stability limit is equal to Transient Stability limit
c) Steady State Stability limit is lee than Transient Stability limit
d) None of the above
Answer
A

4. The stability of the power system is not affected by:
a) Generator reactance
b) Line reactance
c) Excitation of the generator
d) Line losses
Answer
D

5. For stability and economic reason we operate the transmission line with power angle in the range of:
a) 10° to 25°
b) 30° to 45°
c) 60° to 75°
d) 65° to 80°
Answer
B

6. The steady state stability of the power system can be improved by:
a) Using machines of high impedance
b) Connecting transmission line in series
c) Connecting transmission in parallel
d) Reducing the excitation of the machines
Answer
C

7. The transfer of power between two stations is maximum when the phase angle displacement between the voltages of the two stations is
a) Zero

b) 90^{o}
c) 120^{o}
d) 180^{o}
Answer
B

8. The inertia of two group of machines which swing together are M1 and M2. The inertia constant of the system is:
a) M1-M2
b) M1+M2
c) M1M2/(M1+M2)
d) M1/M2
Answer
C

Power System Stability Objective Questions 2

1. The Critical Clearance time of a fault in the power system is related to
a) Reactive power limit
b) Short Circuit limit
c) Steady state stability limit
d) Transient stability limit
Answer
D

2. The equal area criteria of stability is used for:
a) no load on the busbar
b) One machine and infinite busbar
c) More than one machine and infinite busbar
d) None of the above
Answer
B

3. If the torque angle of the alternator increases indefinitely the system will show:
a) Steady state stability limit
b) Transient state stability limit
c) Instability
d) None of the above
Answer
C

4. The steady state stability of the power system can be improved by:
a) Increasing the number of parallel lines between the transmission points
b) Connecting capacitors in series with the line
c) Reducing the excitation of the machines
d) Both a and b
Answer
D

5. The transient stability limit of the power system can be increased by introducing:
a) Series Inductance
b) Shunt Inductance

c) Series Capacitance
d) Shunt Capacitance

C

6. The use of high speed breakers can:
a) Increase the transient stability
b) Decrease the transient stability
c) Increase the steady state stability
d) Decrease the steady state stability

A

7. The inertia constant of the two machines which are not swinging together are M1 and M2. The equivalent inertia constant of the system is:
a) M1-M2
b) M1+M2
c) M1M2/(M1+M2)
d) M1M2/(M1-M2)

A

8. If a generator of 250MVA rating has an inertia constant of 6MJ/MVA, its inertia constant on a 100MVA base is:
a) 15 MJ/MVA
b) 10.5 MJ/MVA
c) 6 MJ/MVA
d) 2,4 MJ/MVA

A

Short Questions & Answers:

UNIT – 1 – THE POWER SYSTEM – AN OVER VIEW AND MODELLING

1. What is single line diagram?
2. What are the components of power system?
.
3. Define per unit value.
4. What is the need for base values?
5. Write the equation for converting the p.u. impedance expressed in one base to another.
6. What are the advantages of per-unit computations?
7. If the reactance in ohms is 15 ohms, find the p.u. value for a base of 15 KVA and 10 KV.
8. A generator rated at 30 MVA, 11 kV has a reactance of 20%. Calculate its p.u. Reactance's for a base of 50 MVA and 10kV.
9. What is impedance and reactance diagram?
10. What are the factors that need to be omitted for an impedance diagram to reduce it to a reactance diagram?
11. What is a bus?
12. What is bus impedance matrix?
13. What are sequence impedance and sequence networks?

PART - B

1. Draw the reactance diagram for the power system shown in Fig.1. Neglect resistance and use a base of 100 MVA, 220 kV in 50 Ω line. The ratings of the generator, motor and transformer are given below.

20. Tutorial Problems

Tutorial class-1

1. Draw the reactance diagram for the power system shown in Fig.1. Neglect resistance and use a base of 100 MVA, 220 kV in 50 Ω line. The ratings of the generator, motor and transformer are given below.

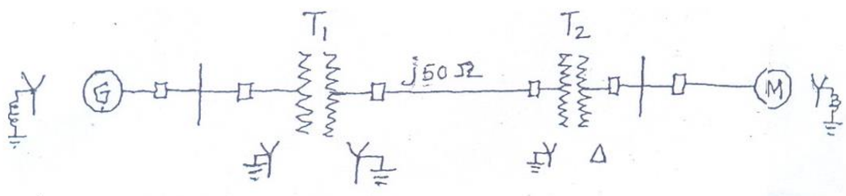


Fig. 1

Generator: 40 MVA, 25 kV, X" = 20%
Synchronous motor : 50 MVA, 11 kV, X" = 30%
Y – Y Transformer : 40 MVA, 33/220 kV, X = 15%
Y - Δ 30 MVA, 11/220 kV, (Δ/Y), X = 15% (16)

2. Draw the structure of an electrical power system and describe the components of the system with typical values (16)
3. Obtain the per unit impedance (reactance) diagram of the power system shown in Fig.3

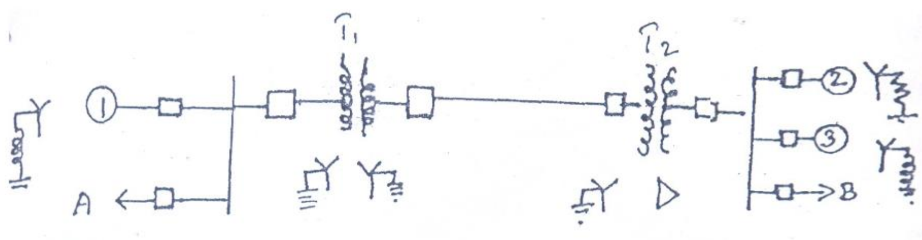
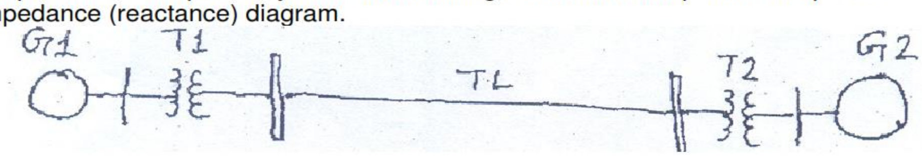


Fig. 3
One-line representation of a simple power system.

Generator No. 1: 30 MVA, 10.5 kV, X" = 1.6 Ohm
Generator No. 2: 15 MVA, 6.6 kV, X" = 1.2 Ohm
Generator No. 3: 25 MVA, 6.6 kV, X" = 0.56 Ohm
Transformer T_1 (3phase) : 15 MVA, 33/11 kV, X = 15.2 Ohm per phase on HT side
Transformer T_2 (3phase) : 15 MVA, 33/6.2 kV, X = 16 Ohm per phase on HT side
Transmission line : 20.5 Ohm/phase
Load A : 15 MW, 11kV, 0.9 p.f. lagging
Load B : 40 MW, 6.6 kV, 0.85 lagging p.f. (16)

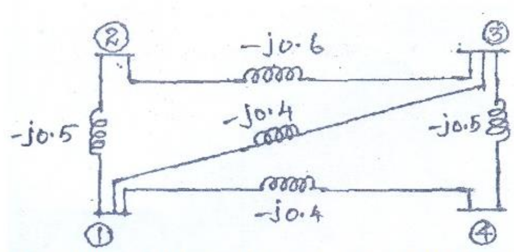
4. Explain the modeling of generator, load, transmission line and transformer for power flow, short circuit and stability studies. (16)
5. Choosing a common base of 20 MVA, compute the per unit impedance (reactance) of the components of the power system shown in Fig.5 and draw the positive sequence impedance (reactance) diagram.



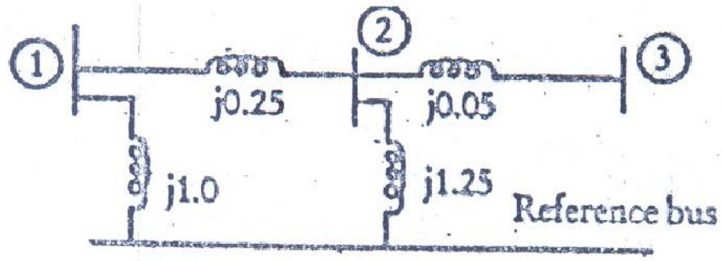
Tutorial Class-2

1. Derive load flow algorithm using Gauss – Seidel method with flow chart and discuss the advantages of the method. (16)
2. Derive load flow algorithm using Newton-Raphson method with flow chart and state the importance of the method. (16)
3. Explain clearly the algorithmic steps for solving load flow equation using Newton – Raphson method (polar form) when the system contains all types of buses. Assume that the generators at the P-V buses have adequate Q Limits. (16)
4. Explain the step by step procedure for the NR method of load flow studies. (16)
5. Find the bus admittance matrix for the given network. Determine the reduced admittance matrix by eliminating node 4. The values are marked in p.u. (16)

Tutorial Class-3



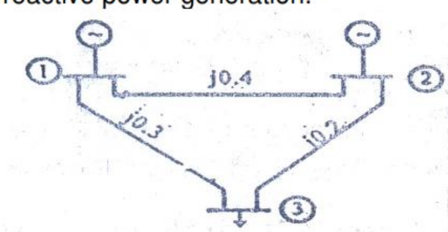
6. Find the bus impedance matrix for the system whose reactance diagram is shown in fig. All the impedances are in p,u. (16)



7. (i) Derive the power flow equation in polar form. (8)
   (ii)Write the advantages and disadvantages of Gauss-Seidel method and Newton-Raphson method. (8)
8. The parameters of a 4-bus system are as under:

Bus code	Line impedance (pu)	Charging admittance (pu)
1-2	0.2 + j 0.8	j 0.02
2-3	0.3 + j 0.9	j 0.03
2-4	0.25 +j 1.0	j 0.04
3-4	0.2 + j 0.8	j 0.02
1-3	0.1 + j0.4	j 0.01

   Draw the network and find bus admittance matrix. (16)
9. With a flow chart, explain the NR Iterative method for solving load flow problem. (16)
10. (i) Compare Gauss-Seidel method and Newton-Raphson method of load flow studies (6)
   (ii) Fig.12 shows a three bus power system.
   Bus 1 : Slack bus, $V= 1.05/0^0$ p.u.
   Bus 2 : PV bus, $V = 1.0$ p.u. $P_g = 3$ p.u.
   Bus 3 : PQ bus, $P_l = 4$ p.u., $Q_l = 2$ p.u.
   Carry out one iteration of load flow solution by Gauss Seidel method. Neglect limits on reactive power generation. (10)



Tutorial Class-4

1. A generator is connected through a transformer to a synchronous motor the sub transient reactance of generator and motor are 0.15 p.u. and 0.35 p.u. respectively. The leakage reactance of the transformer is 0.1 p.u. All the reactances are calculated on a common base. A three phase fault occurs at the terminals of the motor when the terminal voltage of the generator is 0.9 p.u. The output current of generator is 1 p.u. and 0.8 p.f. leading. Find the sub transient current in p.u. in the fault, generator and motor. Use the terminal voltage of generator as reference vector. (16)

2. Explain the step by step procedure for systematic fault analysis using bus impedance matrix. (16)

Tutorial Class-5

3. A 60 MVA, Y connected 11 KV synchronous generator is connected to a 60 MVA, 11/132 KV Δ/Y transformer. The sub transient reactance X''_d of the generator is 0.12 p.u. on a 60 MVA base, while the transformer reactance is 0.1 p.u. on the same base. The generator is unloaded when a symmetrical fault is suddenly placed at point p as shown in Fig. 3 Find the sub transient symmetrical fault current in p.u. amperes and actual amperes on both side of the transformer. Phase to neutral voltage of the generator at no load is 1.0 p.u. (16)

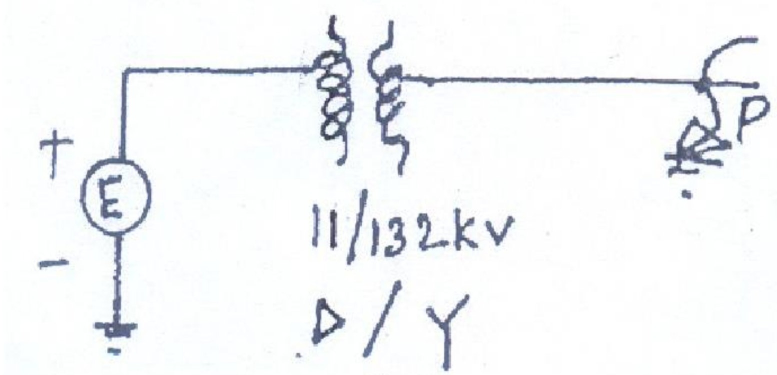


Fig. 3

4. A three –phase transmission line operating at 33 KV and having a resistance and reactance of 5 Ohms and 15 Ohms respectively is connected to the generating station bus-bar through a 5000 KVA step up transformer which has a reactance of 0.05 p.u. Connected to the bus-bars are two alternators, are 10,000 KVA having 0.08 p.u. reactance and another 5000 KVA having 0.06 p.u. reactance. Calculate the KVA at a short circuit fault between phases occurring at the high voltage terminals of the transformers. (16)

21. Known gaps

Known gaps:

As per the industry levels the following are the known gaps of the CMPS subject

Which is in the JNTU curriculum.

1.The CMPS subject as per the curriculum is not matching with the power Systems applications
2.The subject is not matching with real time applications .

Action taken:

22. Group discussion topics.

To be attached.

23.References, Journals, websites and E-links

REFERENCES

1.4.2 Reference Text Books

1. Power System Analysis by Grainger and Stevenson, Tata McGraw Hill.
2. Power System Analysis – by A.R.Bergen, Prentice Hall, Inc.
3. Power System Analysis by Hadi Saadat – TMH Edition.
4. Power System Analysis by B.R.Gupta, Wheeler Publications.

WEBSITES

1. www.electricalengineernetbase/tutorial/
2. www.electricaldrives.com
3. www.ieee.org/t**utorials**
4. www.mit.edu/electricalengineer.com

24.Quality Control Sheets

To be attached

25. STUDENT LIST

Class / Section: EEE 3yr/1Isem

26.Group-Wise students list for discussion topics

GROUP1

To be attached

GROUP 2

GROUP 4

GROUP 5

GROUP 6

GROUP 7

GROUP 8
GROUP 9

GROUP 10

Closure Report:

1. Total Number of classes planned - 60
2. Total Number of classes actually taken - 75
3. Total Number of students attended for the internal exam -
4. Total Number of students attended for the external exam -
5. Total number of students passed the exam -
6. Pass percentage -